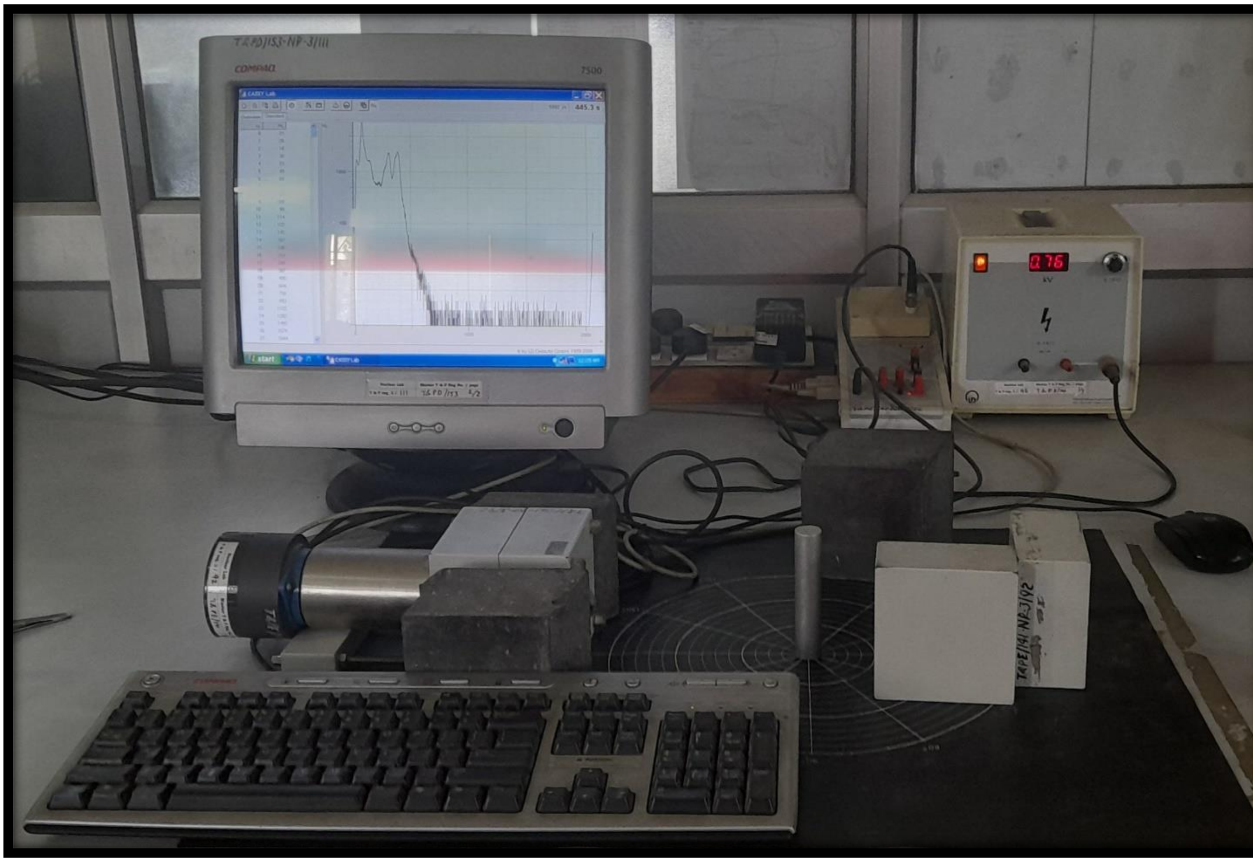
Nuclear Physics Lab
Lab Manual



Compton Scattering and Gamma-Ray Spectroscopy

Aim:

To study the Compton equations.

Apparatus Used:

NaI(TI) detector, Highly Active Radio Nuclide Source ^{137}Cs, Computer system, High Voltage Power Supply, Aluminum Rod (Scattering element), Lead Bricks.

Theory:

The Compton effect is the quantum theory of the scattering of electromagnetic waves by a charged particle in which a portion of the energy of the electromagnetic wave is given to the charged particle in an elastic, relativistic collision. Compton scattering was discovered in 1922 by Arthur H. Compton (1892-1962) while researching the scattering of X-rays with high elements.

Compton scattering is the main focus of this experiment, but it is necessary to understand the interactions of high energy, electromagnetic photon radiation with materials in general. Gamma rays are high-energy photons emitted from radioactive sources. When they interact with matter, there are three primary ways their energies can be absorbed by materials. There is the photoelectric effect, Compton scattering and pair production. In addition to these primary processes, there are several lesser ways such as X-ray production and Bremsstrahlung. The Compton Effect is studied with the measurement of gamma-ray energy using a scintillator, photomultiplier tube and multichannel analyzer.

Compton scattering involves the scattering of photons by charged particles where both energy and momentum are transferred to the charged particle while the photon moves off with reduced energy and a change of momentum. Generally, the charged particle is an electron considered to be at rest and the photon is usually considered to be an energetic photon such as an X-ray photon or gamma ray photon. In this experiment gamma rays from a caesium-137 source are used for the source of photons that are scattered and each photon has an energy of 0.662 MeV when incident on the target scatterer. The charged particle is assumed to be an electron at rest in the target. While the theory here is applied to gamma rays and electrons, the theory works just as well for less energetic photons such as those found in visible light and other particles.

The theory of Compton scattering uses relativistic mechanics for two reasons. First, it involves the scattering of massless photons, and secondly, the energy transferred to the electron is comparable to its rest energy. As a result, the energy and momentum of the photons and electrons must be expressed using their relativistic values. The laws of conservation of energy and conservation of momentum are then used with these relativistic values to develop the theory of Compton scattering.

From the special theory of relativity, an object whose rest mass is m_o and is moving at a velocity *v* will have a relativistic mass m given by

$$m = \frac{mo}{\sqrt{1-\frac{V^2}{c^2}}} \quad (1)$$

Relativistic momentum p is defined mv so that squaring of equation (1) and multiplying c^4 leads to

$$m^2 - m^2(v^2/c^2) = m_0^2$$

$$m^2c^4 - m^2v^2c^2 = m_0^2c^4$$

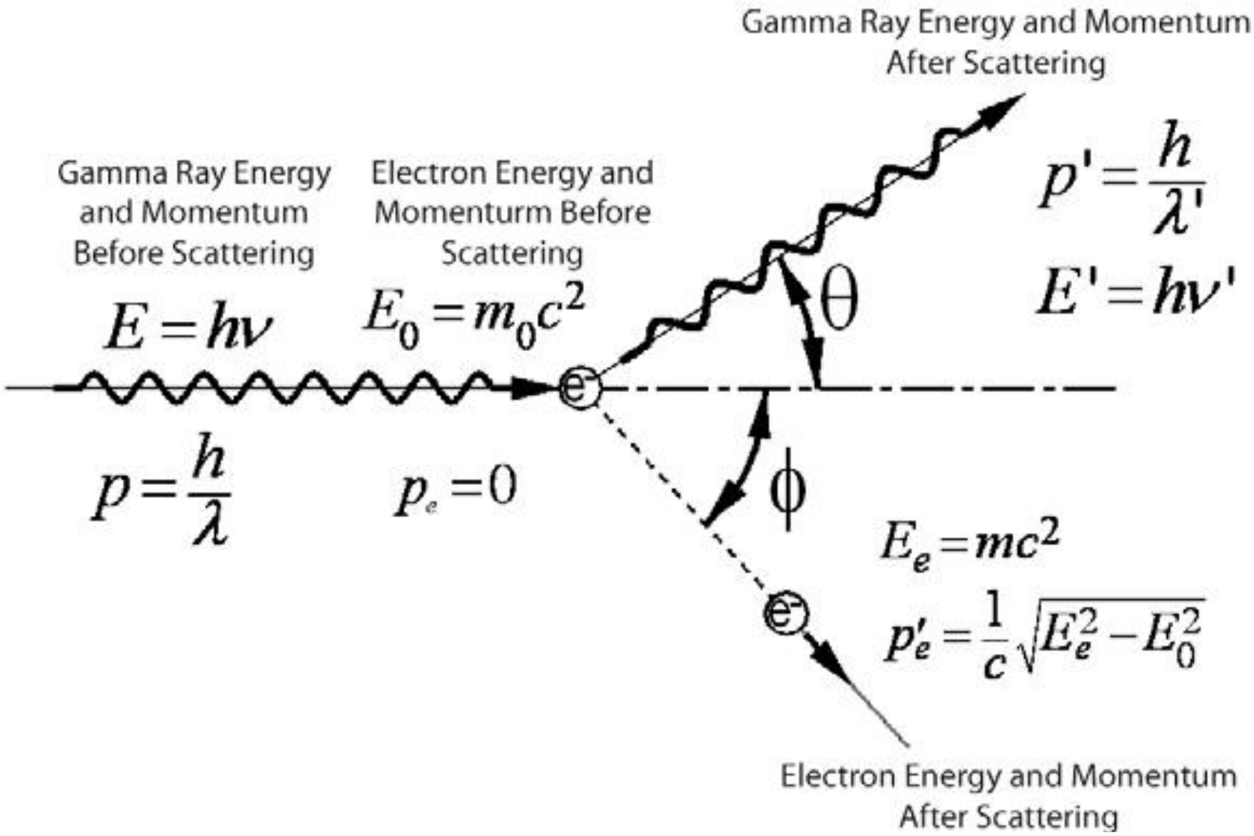
$$E^2 - (pc)^2 = E_0^2$$

$$(pc)^2 = E^2 - E_0^2 \quad (2)$$

Equation (2) then relates the magnitude of the relativistic momentum p of an object to its relativistic energy E and its rest energy E_0. From this equation, it is readily seen that the magnitudes of momentum and energy of a massless particle such as a photon are related by

$$pc = E \quad (3)$$
$$p = E/c \quad (4)$$

Figure 1 illustrates the scattering of an incident photon of energy E = hν moving to the right in the positive x direction with a momentum p = h/λ and interacting with an electron at rest with a momentum $p_e = 0$ and energy equal to its rest energy, $E_0 = m_oc^2$. The symbols ν and λ are the standard symbols used for Planck's constant, the photons frequency and its wavelength. mo is the rest mass of the electron. In the interaction, the gamma-ray scattered in the positive x and y direction at an angle θ with the momentum of magnitude p′ = hν′/c = h/λ′ and energy E′ = hν′.



From the law of conservation of energy, the energy of the incident gamma ray, hν, and the rest mass of the electron, E_0, before scattering is equal to the energy of the scattered gamma ray, h'ν, and the total energy of the electron, E_e, after scattering, or

$$h\nu = h'\nu + E_e \quad (5)$$

From equation (2), the relationship between the total energy, E_e, of the electron after scattering, its rest mass energy, E_0, and its relativistic momentum, p_e, is given by

$$E_e^2 = (p_e^{'}c)^2 + E_0^2 \qquad (6)$$

$$E_e = \sqrt{(p_e^{'}c)^2 + E_0^2} \quad (7)$$

Substituting Equation (7) into Equation (5) yields

$$h\nu + E_0 = h\nu' + \sqrt{(p_e^{'}c)^2 + E_0^2} \qquad (8)$$

Using the relationship between the energy of a photon (massless particle) and its momentum from equation (4) gives

$$pc + E_0 = p'c + \sqrt{(p'_e c)^2 + E_0^2} \qquad (9)$$

Rearranging gives

$$(p - p')c + E_0 + \sqrt{(p'_e c)^2 + E_0^2} \qquad (10)$$

And

$$(p - p')\ c + E_0 + 2(p - p')\ cE_0 = (p'_e c) + E_0^2 \quad (11)$$

That results in the following expression based on the conservation of energy

$$p^2 + p'^2 - 2pp' + 2(p - p')\ E_0/C = p'^2_e \qquad (12)$$

Equation (12) is then an expression relating the momentum p_e of the electron given to it by a scattered gamma ray whose initial momentum was p and whose final momentum is p. The electron was assumed to be initially at rest and it was also assumed to be given enough energy for relativistic mechanics to apply. Equation (12) is solely based on the law of conservation of energy, but another independent expression for the momentum p_e can be found based on the law of conservation of momentum.

In the scattering, process momentum must be conserved so that

Total momentum Before = Total momentum After. (13)

Since momentum is a vector quantity.

Total momentum in X-direction Before = Total Momentum in X-direction After (14)

And

Total momentum in Y-direction Before = Total Momentum in Y-direction After (15)
For an electron at rest, its initial momentum is zero and has no x and y components. For an incident gamma ray photon moving in the positive x direction and interacting with an electron at rest, the initial x-component of momentum is p and the y-component is zero so that

$$p = p'\cos\theta + p'_e\cos\phi \qquad (16)$$

$$0 = p'\sin\theta + (-)\ p'_e\sin\phi \quad (17)$$

Where p′ and p_e' are the momenta of the scattered gamma ray and electron after interacting. Rearranging Equations (16) and (17) and squaring both sides of each produce

$$p_e' \cos\phi = p - p' \cos\theta \quad (18)$$

$$p_e' \sin\phi = p' \sin\theta \quad (19)$$

$$p_e'^2 \cos^2\phi = p'^2 + p'^2 \cos^2\theta - 2pp' \cos\theta. \quad (20)$$

$$p_e'^2 \sin^2\phi = p'^2 \sin^2\theta \quad (21)$$

Adding equations (20) and (21) yields

$$p_e'^2 (\sin^2\phi + \cos^2\phi) = p'^2 + p'^2 (\sin^2\theta + \cos^2\theta) - 2pp' \cos\theta. \quad (22)$$

That can be simplified using the identity $\sin^2 x + \cos^2 x = 1$ to further yield

$$p_e'^2 = p'^2 + p'^2 - 2pp' \cos\theta. \quad (23)$$

Equation (23) is then expression based on the law of conservation of momentum that relates the momentum given to the electron from its rest position by the incident gamma ray of momentum p interacting with the electron so that it is scattered off at angle θ with momentum p′.

Equating equations (12) and (23), one based on conservation of energy and the other on conservation of momentum gives

$$p'^2 + p'^2 - 2pp' + 2(p - p') E_0/c = p'^2 + p'^2 - 2pp' \cos\theta \quad (24)$$

then reduces to

$$2(p - p') E_0/c = 2pp' - 2 pp' - \cos\theta \quad (25)$$

And

$$\frac{1}{p'} - \frac{1}{p} = \frac{C}{E_0} (1 - \cos\theta) \quad (26)$$

Using the relationship for momentum, energy, wavelength and frequency for photons, $p = h/\lambda = h\nu/c = E/c$, equation (26) can be transformed into

$$\frac{1}{E'} - \frac{1}{E} = \frac{1}{E_0}(1-\cos\theta) \qquad (27)$$

Where E_0 is the rest mass energy of the electron.

That relates the energy of a scattered photon E′ to the energy of the incident photon E and the scattering angle θ.

Equation (27) is a simple equation that can be used to verify the theory for the Compton Effect. The energy of incident gamma ray E can be easily measured with a scintillator-photomultiplier detector and multichannel analyzer system., The energy of the scattered gamma rays E′ as a function of θ can also be easily measured with the same system. A plot of measurement of $\frac{1}{E'} - \frac{1}{E}$ versus measurements of [1-cosθ] should result in a linear graph whose slope is the inverse of the electron's rest energy.

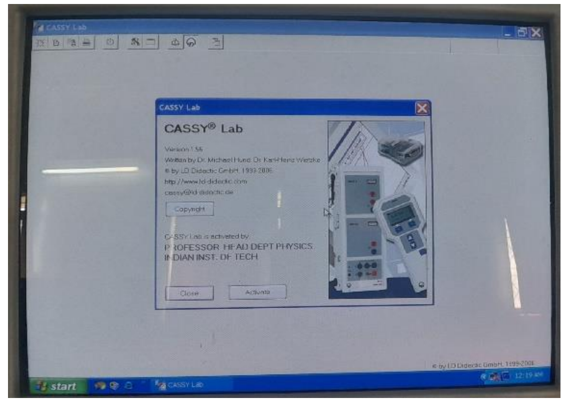
Gamma-ray spectroscopy and the Scintillation detector

Data to verify the Compton scattering theory is collected in this experiment using a gamma ray spectrometer that consists of a scintillation detector, high voltage supply and a multichannel analyzer to measure the energy distribution of the detected gamma rays. There are many ways to detect gamma rays and these include ionization chambers, photographic film, proportional counters, Geiger Muller detectors, solid state detectors, germanium detectors, liquid and solid scintillation materials with photomultipliers tubes and several methods using similar materials and approaches.

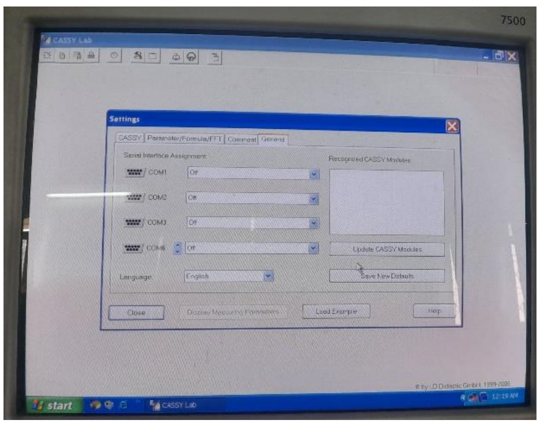
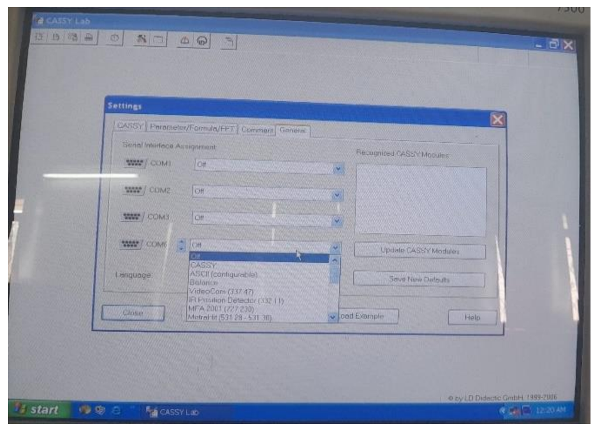
To study the Compton Effect a gamma ray spectroscopy method is needed to measure the gamma ray's energy before and after an interaction. A scintillation detector is capable of doing this and one used in this experiment is composed of sodium iodide (NaI) scintillation crystal and a photomultiplier tube. The detector system produces a voltage pulse that is proportional to the energy deposited in the crystal by the absorbed gamma ray. The detected gamma ray may be from the radioactive source either directly or by scattering. The size of the voltage pulse and hence the energy deposited in the detector is measured with a multichannel analyzer (MCA). The energy deposited in the scintillation crystals depends on the type of interaction between the gamma-ray and the crystal even for a single gamma-ray of a single energy. An MCA measures the distribution of voltage pulse heights, or spectrum of voltage pulses, for multiple gamma rays interacting in the crystal depending on the type of interaction that occurs.

Procedure:

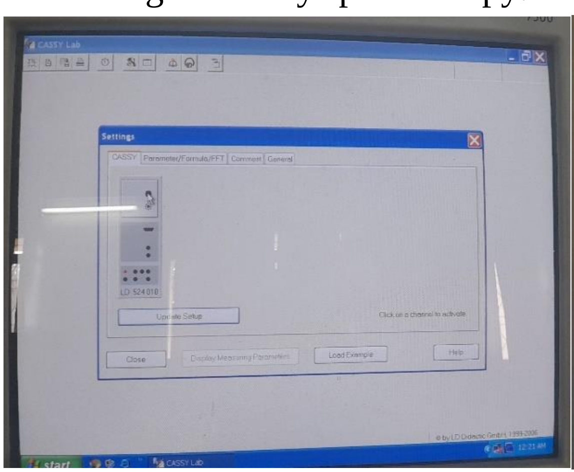
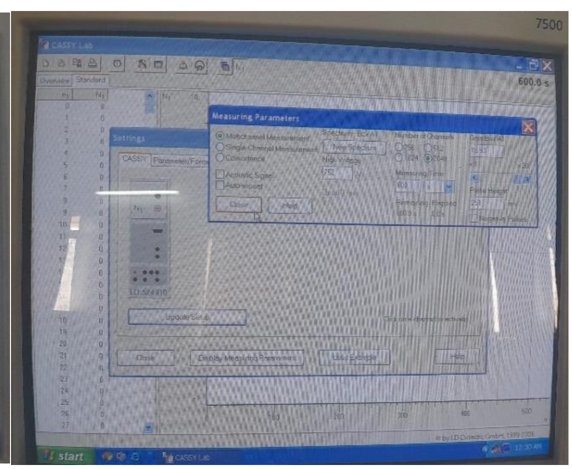
1. Put ^{137}Cs source close to NaI(TI) detector.
2. Set the operating voltage of the NaI (TI) detector to 760 volts.
3. Open the CASSYMCA software, a new tab is open and click on the close button to close it.

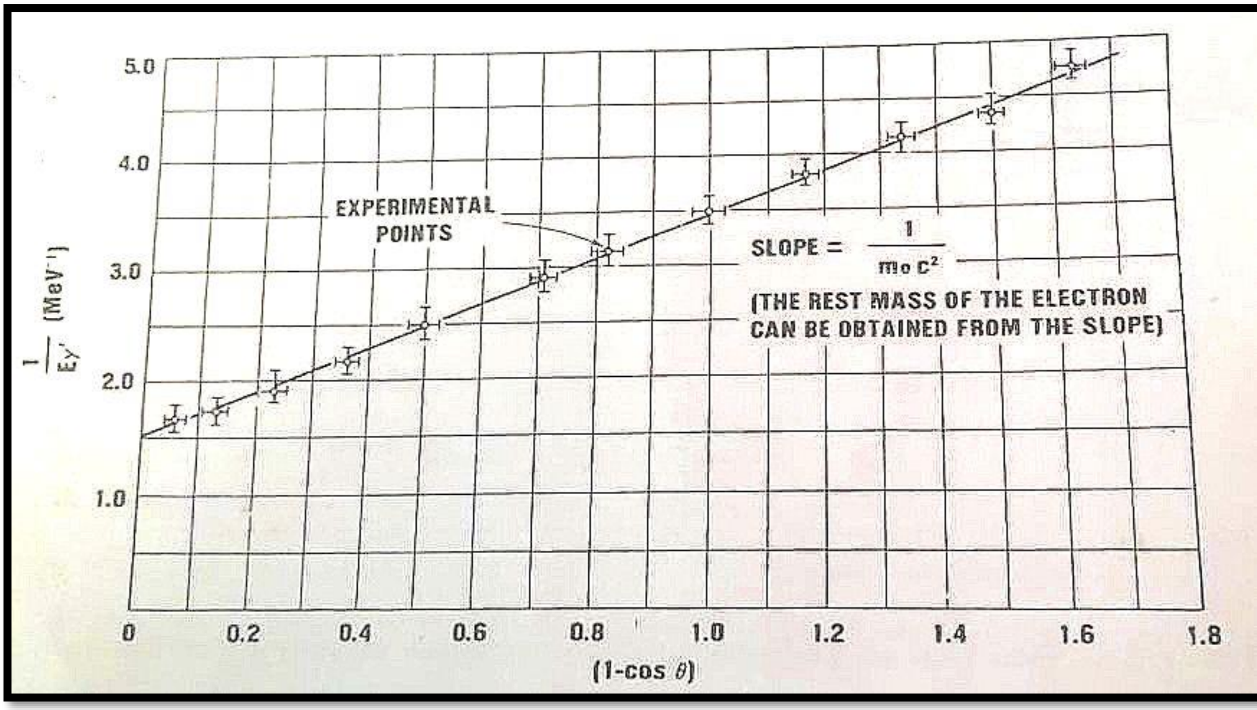
4. Again a new tab is open, click on COM6 and select the CASSY option, then go to the CASSY tab option and click on it.

5. Two dots appear on a new tab when you click on the CASSY option. Click on the first dot, and fill in all the experimental requirements to measure the spectrum of gamma-ray spectroscopy.

6. Record the spectrum for 600 secs.
7. Put an aluminium rod in between the source and detector.
8. Keep the detector at rest, the distance between the aluminium rod and detector is 15cm and vary the scattering angle between incident photon and scattered photon.
9. Record the spectra at different scattering angles for 600 sec.
10. Make a table, write scattering angles, and theoretically calculate scattered photon energy for different scattering angles in a table.
11. Note down channel no. for different scattering angles.
12. Measure the size of the detector and calculate the activity of the source using the decay scheme of the source.
13. Calculate the counts in the energy spectrum using origin8 software.
14. Compare the experimental scattered photon energy with theoretically scattered photon energy and report the error in your results after the calculations.
15. Plot a graph in between $\frac{1}{E'}$ **vs (1-cosθ)** as shown below**,** calculate the slope of the graph and compare it with the theoretically calculated slope.



16. Show all calculations with errors in the result section.

Calculation:

$$\frac{1}{E'} = \frac{1}{E} + \frac{1}{E_0}(1-\cos\theta) \quad (28)$$

The equation in the form of y = mx +b

$$y = \frac{1}{E'} ,$$

$$m = \text{slope} = \frac{1}{m_0 c^2} = \frac{1}{0.511\ \text{Mev}} = 1.96\ \text{Mev}^{-1}$$

$$b = \frac{1}{E} = 1.51\ \text{Mev}^{-1}$$

Table:

S.No.	Peak Channel	θ Degrees	(1-Cosθ)	E'(Calculated)	E'(Measured)	$\frac{1}{E'}$ Mev^{-1}
1.						
2.						
3.						
4.						
5.						
6.						
7.						

Result:

Precaution and sources of error:

1. Detector is very costly, do not apply high voltage to the detector. It can burn the detector.
2. Operating voltage of the detector increase in the step by volts.
3. Do not play with the source.
4. Do not touch the source with your hands, always use a tweezer.

Courtesy: Rahul Chauhan
(Research Scholar)