CORNU'S METHOD

OBJECT:

To determine Young's modulus and Poisson's ratio of glass by Cornu's method.

APPARATUS:

Wooden stand to carry the experimental glass beam, a small rectangular glass plate and another glass plate inclined at an angle of 45^o to the incident beam, travelling Vernier microscope, sodium light, hangers, weights, screw gauge, Vernier calipers, etc.

PRINCIPLE OF THE EXPRIMENT:

If a rectangular glass beam is deformed in the form of a curvature under the action of bending moment, the longitudinal as well as the transverse filaments of the beam on either side of the neutral surface change in length. In Cornu's method the curvature is produced by supporting the glass beam on two knife edges placed near the ends of the beam with their edges normal to the axis of the beam, and suspending weighs from the ends. A plane cover-glass rests on the curved beam to form an air-film between the two. On illuminating the air-film with sodium light we obtain two conjugate systems of hyperbolic interference fringes round the point of contact of two glass plates as shown in Fig.5. By measuring longitudinal curvature, Y can be determined. In addition to longitudinal, if we measure transverse curvature, we can find Poisson's ratio (σ).

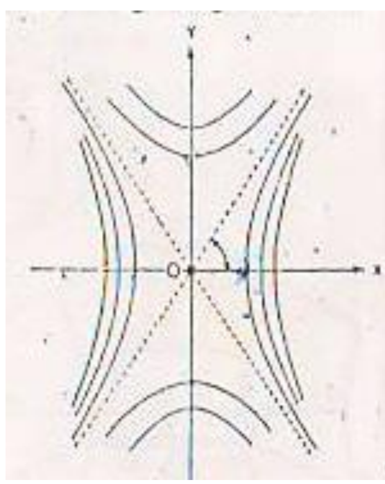

Fig.1

DESCRIPTION OF THE APPARATUS:

The experimental glass beam *AB*, which is of about 50 cm length, 3 cm breadth and 1.5mm thickness; is supported on two knife edges K_1 and K_2 nearly 20cm apart. Two hangers carried by light metallic frame, or thread loops, are suspended symmetrically near both ends. In the middle of the beam is placed a square (or rectangular) glass plate *P* of thickness of about 2mm, each side of the square being about 3mm. Light from a monochromatic source (Sodium Lamp) is made to fall on the cover glass plate *P* and the beam *AB* by means of another glass plate *G* arranged at an angle of 45° with the horizontal. The interference hyperbolic fringes, formed between the lower face of the cover plate and the upper curved surface of the beam can be views in a travelling microscope *M*. A plane mirror M_1 suitably arrange at about 45° to the horizontal directs the light into the microscope.

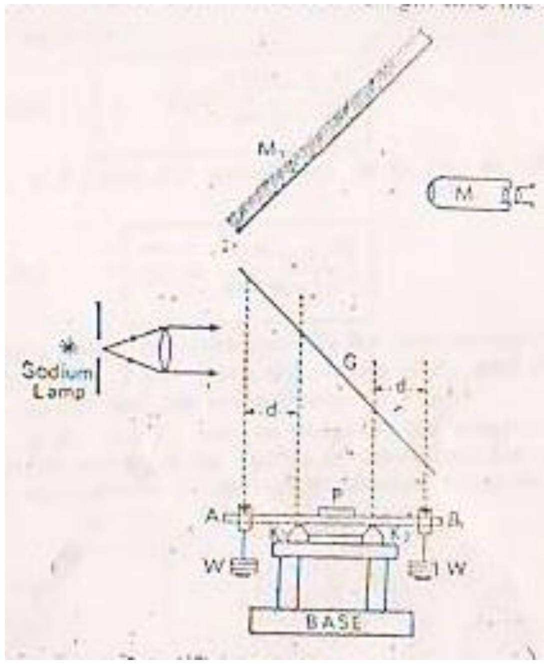

Fig. 2

THEORY AND DERIVATION OF FORMULA:

The experimental beam AB is bent under the action of the loads W, W hanging at both the ends. The bending moment G acting at all the transverse sections of the central span is related to the longitudinal radius of curvature R_1 by:

$$G = \frac{YI}{R_1}, \qquad \ldots(1)$$

Where Y is the Young's modulus for the material of the beam and *I* is the geometrical moment of inertia of cross-section of the beam about an axis passing through its centroid and perpendicular to the plane of bending. If *b* is the breadth and *t* the thickness of the beam, then

$$I = \frac{1}{12} bt^3 \qquad \ldots(2)$$

If *d* is the distance between the point of support of the load *W* and knife-edge nearer to it, then

$$G = Wd \qquad \ldots(3)$$

From eq. (1) and (2)

$$\frac{YI}{R_1} = Wd$$

If the beam has an initial radius of curvature R_o due to its own weight, then

$$\frac{Y1}{R_1} - \frac{Y1}{R_o} = Wd \qquad \ldots(4)$$

If for a load W' the radius of curvature (longitudinal) of the beam is R_1', then in anology to eq.(4), we have

$$\frac{Y1}{R_1'} - \frac{Y1}{R_o} = W'd \qquad \ldots(5)$$

Substituting the value of I from eq. (2), we have

$$Y1 = \left[\frac{1}{R_1} - \frac{1}{R_1'}\right] = (W - W')d$$

$$\therefore \; Y = \frac{(W-W')d}{I\left(\frac{1}{R_1} - \frac{1}{R_1'}\right)} \qquad \ldots(6)$$

Substituting the value of I from eq. (2), we have

Formula 1st $\quad Y = \frac{1(W-W')d}{bt^3\left(\frac{1}{R_1} - \frac{1}{R_1'}\right)} \qquad \ldots(7)$

In order to determine R_1 and R_1'; let us refer to Fig. 3. Let *MN* be the lower surface of the cover plate and *AB* the curved section of the upper surface of the curved beam for weight *W*. Let O_1 be the centre of the fringe-system and *A, B* the positions of the 1st pair of fringes in the longitudinal direction (along X-axis). Let $AB = X_1$. From the geometry of the figure, we have

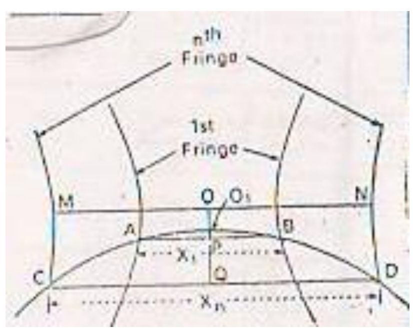

Fig. 3

$$(2R_1 - O_1P)\, O_1P = \left(\frac{X_1}{2}\right)^2$$

But $O_1P \ll 2R_1$, hence neglecting $(O_1P)^2$, we have

$$2R_1\,(O_1P) = \left(\frac{X_1}{2}\right)^2$$

Similarly, if C and D represent the positions of nth pair of fringes, separated by X_r, then

$$2R_1\,(O_1Q) = \left(\frac{X_n}{2}\right)^2$$

Subtracting these two equations, we have

$$2R_1(O_1Q - O_1P) = \frac{1}{4}(X_n^2 - X_1^2).$$

But $(O_1Q - O_1P) = \frac{(n-1)\lambda}{2}$ for a bright fringe, where λ is the wave-length of light used.

Formula 2nd

$$\therefore\ R_1 = \left(\frac{(X_n^2 - X_1^2)}{4\lambda(n-1)}\right) \qquad \ldots(19)$$

Similarly, if X_n' and X_1'correspond to R_1' i.e., to weight W' then

Formula 3rd

$$R_1' = \frac{(X_n'^2 - X_1'^2)}{4\lambda(n-1)} \qquad \ldots(20)$$

The formula Ist is requite formula for determining the value of Young's modulus Y for glass, the value of R_1 and R_1' being given by formula 2nd and 3rd respectively.

Similarly, if Y_n and Y_1refer to fringes in the transverse direction (along Y-axis) and $R_1$is the radius of anti-elastic curvature)as there will be contraction in the perpendicular direction of the beam) then

Formula 4th

$$R_1 = \left(\frac{(Y_n^2 - Y_1^2)}{4\lambda(n-1)}\right) \qquad \ldots(21)$$

Hence the Position's ratio (σ) is given by

$$\sigma = \frac{longitudinal\ radius\ of\ curvature}{transverse\ radius\ of\ curvature} = \frac{R_1}{R_1}$$

Formula 5th

$$\therefore\ \sigma = \frac{(X_n^2 - X_1^2)}{(Y_n^2 - Y_1^2)} \qquad \ldots(22)$$

This is required formula for determining the Poisson's ratio σ.

PROCEDURE:

(1) The experiment is arranged as shown in Fig.6. Weights of certain mass, say 200gm are placed on each hanger at both the ends of the experimental beam AB. The air-film between the cover plate P and the curved experimental beam is illuminated by means

of an extended source of monochromatic light (sodium lamp), so that hyperbolic fringes are formed. A travelling microscope is focused on the fringes.

(2) The distance between the first pair of fringes and the nth (say 10th) pair of fringes is determined in the longitudinal direction by means of the microscope. These readings give X_1 and X_n respectively. Similarly, by measuring the distances in the transverse direction, Y_1 and Y_n are determined. The Poisson's ratio σ is the calculated from $\sigma = (X_n^2 - X_1^2)/(Y_n^2 - Y_1^2)$.

(3) Different sets of reading X_n, X_1, Y_n and Y_1 are taken from different weights. Finally the mean value of σ is determined.

(4) The values of R_1 and R_1' are obtained by means of eq. (19) and (20), using the values of X_n, X_1, X_n' and X_1' as measured above in two sets for weights W and W' (say 200 and 250gm). Here n and 10 and λ for sodium light is 5893 X 10^{-8}cm.

(5) The thickness t of the experimental beam is measured by the screw gauge and breadth 'b' by a Vernier calipers. To measure the distance d, we measure the distance of the knife-edges K_1 and K_2 from their respective ends A and B (Fig.6), and the average of the two readings is taken.

(6) On substituting the values of R_1, R_1'(as determined above), W(= 250g), W'(= 250g), the constants b, t and d in eq.

OBSERVATIOINS AND CALCULATIONS:

Least count of the Vernier of travelling microscope = …cm.

(1) Distance measured in longitudinal direction [$n = 10$].

Set No.	Weight (dynes)	No. of fringe	Distance (Diameter)			(Distance) i.e. $(X)^1$ (cm)
			Reading on left (cm)	Reading of right (cm)	Difference X (cm)	
1st	W (= 200g)	1st nth (10th)	… … … …	… … … …	$X_1 = \dots$ $X_n = \dots$	$(X_1)^2 = \dots$ $(X_n)^2 = \dots$
2nd	W' (=250g)	1st nth (10th)	… … … …	… … … …	$X_1' = \dots$ $X_n' = \dots$	$(X_1')^2 = \dots$ $(X_n')^2 = \dots$
3rd	W' (=300g)	1st nth (10th)	… … … …	… … … …	$X_1' = \dots$ $X_n' = \dots$	$(X_1')^2 = \dots$ $(X_n')^2 = \dots$
4th	W' (=350g)	1st nth (10th)	… … … …	… … … …	$X_1' = \dots$ $X_n' = \dots$	$(X_1')^2 = \dots$ $(X_n')^2 = \dots$

$$\therefore\ R_1 = \left(\frac{(X_n^2 - X_1^2)}{4\lambda(n-1)}\right)$$

or

$$\frac{1}{R_1} = \frac{4\lambda(n-1)}{X_n^2 - X_1^2} = \cdots cm^{-1} \text{ for 200 g.}$$

Similarly

$$\frac{1}{R_1'} = \frac{4\lambda(n-1)}{X_n'^2 - X_1'^2} = \cdots cm^{-1} \text{ for 250 g.}$$

$$\frac{1}{R_1'} = \frac{4\lambda(n-1)}{X_n''^2 - X_1''^2} = \cdots cm^{-1} \text{ for 300 g.}$$

and

$$\frac{1}{R_1'} = \frac{4\lambda(n-1)}{X_n'''^2 - X_1'''^2} = \cdots cm^{-1} \text{ for 350 g.}$$

(2) Distances measured in transverse direction $[n = 10]$

Set No.	Weight (dynes)	No. of fringe	Distance (Diameter)			(Distance) i.e. $(Y)^1$ (cm)
			Reading on left (cm)	Reading of right (cm)	Difference Y (cm)	
1st	W (= 200g)	1st nth (10th)	… … … …	… … … …	$Y_1 = \ldots$ $Y_n = \ldots$	$(Y_1)^2 = \ldots$ $(Y_n)^2 = \ldots$
2nd	W' (=250g)	1st nth (10th)	… … … …	… … … …	$Y_1' = \ldots$ $Y_n' = \ldots$	$(Y_1')^2 = \ldots$ $(Y_n')^2 = \ldots$
3rd	W' (=300g)	1st nth (10th)	… … … …	… … … …	$Y_1'' = \ldots$ $Y_n'' = \ldots$	$(Y_1'')^2 = \ldots$ $(Y_n'')^2 = \ldots$
4th	W' (=350g)	1st nth (10th)	… … … …	… … … …	$Y_1''' = \ldots$ $Y_n''' = \ldots$	$(Y_1''')^2 = \ldots$ $(Y_n''')^2 = \ldots$

$$\therefore\ R_1 = \left(\frac{(Y_n^2 - Y_1^2)}{4\lambda(n-1)}\right)$$

or

$$\frac{1}{R_1} = \frac{4\lambda(n-1)}{Y_n^2 - Y_1^2} = \cdots cm^{-1} \text{ for 200 g.}$$

Similarly

$$\frac{1}{R_1'} = \frac{4\lambda(n-1)}{Y_n'^2 - Y_1'^2} = \cdots cm^{-1} \text{ for 250 g.}$$

$$\frac{1}{R_1'} = \frac{4\lambda(n-1)}{Y_n''^2 - Y_1''^2} = \cdots cm^{-1} \text{ for 300 g.}$$

and

$$\frac{1}{R_1'} = \frac{4\lambda(n-1)}{Y_n'''^2 - Y_1'''^2} = \cdots cm^{-1} \text{ for 350 g.}$$

(3) Calculation of Poisson's ratio σ :

Set. No.	Poisson's ratio σ	Mean σ
1st	$\sigma = \frac{R_1}{R_1} = \frac{X_n^2 - X_1^2}{Y_n^2 - Y_1^2} = \cdots$	

2nd	$\sigma = \frac{R'_1}{R'_1} = \frac{X'^2_n - X'^2_1}{Y'^2_{\prime n} - Y'^2_{\prime 1}} = \cdots$	
3rd	$\sigma = \frac{R''_1}{R''_1} = \frac{X''^2_n - X''^2_1}{Y''^2_n - Y''^2_1} = \cdots$	
4th	$\sigma = \frac{R'''_1}{R'''_1} = \frac{X'''^2_n - X'''^2_1}{Y'''^2_n - Y'''^2_1} = \cdots$	

(4) Constants of the Apparatus:

L.C. of screw gauge = …cm

L.C. of Vernier calipers = …cm.

(i) Thickness of the beam (1) : (By screw gauge) for W = 0.

S.No.	Zero error (cm)	Main scale reading (cm)	Circular scale reading (cm)	Total reading (cm)	Mean l (cm)

(ii) Breadth of the beam (b) : (By Vernier calipers) for W = 0

S.No.	Zero error (cm)	Main scale reading (cm)	Vernier scale reading (cm)	Total reading (cm)	Mean l (cm)

(iii) Distance (d) : (By Vernier calipers) for W = 0

S.No.	Distance K_1A (cm)	Distance K_2B (cm)	$\therefore d = \frac{K_1A - K_2B}{2}$ (cm)	Mean σ

(5) Calculations for the Young's modulus:
Λ for sodium light = 5893 x 10^{-3} cm.

Set 1st

$$Y = \frac{12(W-W')d}{bt^3\left[\frac{1}{R_1}-\frac{1}{R_1'}\right]}$$, where W = 200g and W' = 250g

= … x 10^{11} dynes/cm^2.

Set 2nd

$$Y = \frac{12(W''-W''')d}{bt^3\left[\frac{1}{R_1''}-\frac{1}{R_1'''}\right]}$$, where W'' = 3000g and W''' = 350g

= … x 10^{11} dynes/cm^2.

[Similarly, take other pairs e.g. $(W-W')$, $(W-W''')$, $(W'-W'')$, $(W'-W''')$ and find the value of Y for each set].

Mean value of Y = …x 10^{11} dynes/cm^2.

RESULTS:

(1) Poisson's ratio (σ) for glass = …
(2) Young's Modulus (Y) for glass = …x 10^{11} dynes/cm^2.

PRECAUTIONS AND SOURCES OF ERROR:

(1) The beam should be placed on the knife –edges symmetrically (i.e. K_1A and K_2B must be equal).
(2) The constants of the apparatus should be determined only when the beam is unloaded.
(3) The beat beam should not touch the wooden stand.
(4) Al the glass plates must be plane (optically) and clean.
(5) If the first few fringes are blurred then we take the reading for s (say 4th) and n + s (say11); and Y, σ are determined by using the relations. $R_1 = \frac{X_{n+1^2-Y_1^2}}{4\lambda n}$ and $R_1 = \frac{Y_{n+1^2-Y_1^2}}{4\lambda n}$
(6) To eliminate effects arising due to imperfect flatness in the experimental beam and cover plate, the beam is turned over and the loads applied as usual. It is however necessary to place the cover plate below the beam and bring their surfaces together with the help.