Young modulus by Cornu's Method

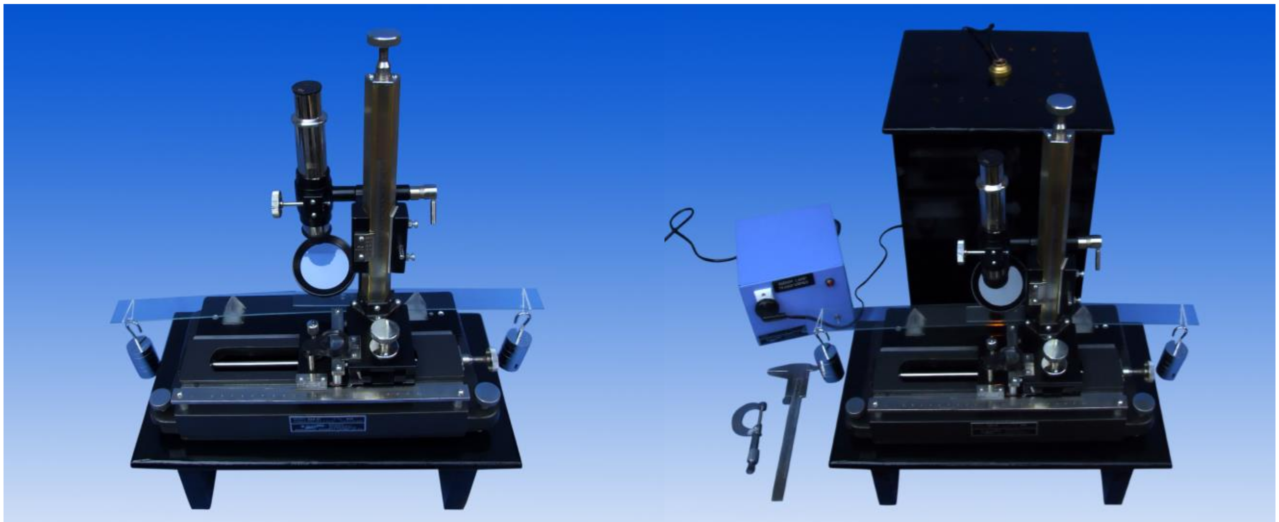

Fig. 1 (Complete Cornu's Set-up)

Object: To determine Young's modulus and Poisson's ratio of glass by Cornu's method.

Apparatus used: Traveling microscope with X-Y-Z motion specially designed for Cornu's experiment. Cornu's assembly fitted with glass plate inclined at an desired angle say 45^0 in this case, The experimental glass beam, A small rectangular glass plate, Set of 50 gm weight with hanger, Sodium Vapor lamp, screw gauge ,Vernier Callipers. Wooden plane to level the Cornu's set-up with light source.

Principle of the Experiment: If a rectangular glass beam is deformed in the form of a curvature under the action of bending moment, the longitudinal as well as the transverse filaments of the beam on the either side of the neutral surface change in length. In this method the curvature is produced by supporting the glass beam on two knife edges placed near the ends of the beam with their edges normal to the axis of the beam, and suspending weights from the ends.

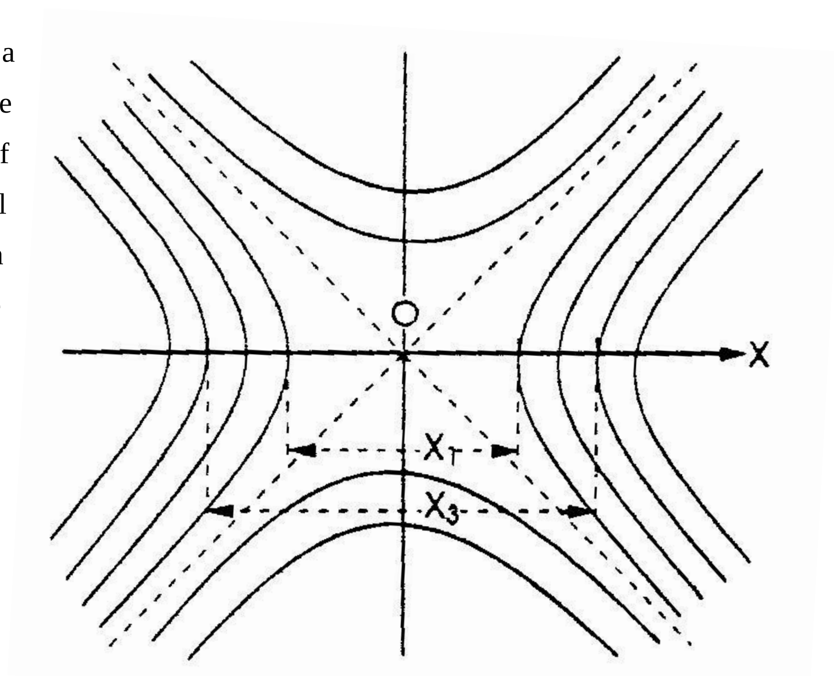

Fig. 2

A plane small glass plate rests on the curved beam to form an air film between the two plates. On illuminating the air film with sodium light we obtain two conjugate systems of hyperbolic interference fringes round the point of contact of two glass plates as shown in fig 2. By measuring longitudinal curvature, Y can be determined. In addition to longitudinal, if we measure transverse curvature, we can find Poisson's ratio (σ).

Description of the apparatus:

The experimental glass beam AB is supported on two knife edges made of glass K_1 and K_2 as in Fig. 3. Two hangers made of thread loops are suspended symmetrically near both ends of the glass plate. A small rectangular glass plate P is placed in the middle of the beam AB. Light from a monochromatic source (sodium lamp) is made to fall on the cover glass plate P and the beam AB by means of the another glass plate G arranged at an angle of 45^0 with the horizontal such that the parallel beam is reflected from the lower surface. The interference hyperbolic fringes formed between the lower face of the cover plate and the upper curved surface of the beam can be viewed in a traveling microscope M.

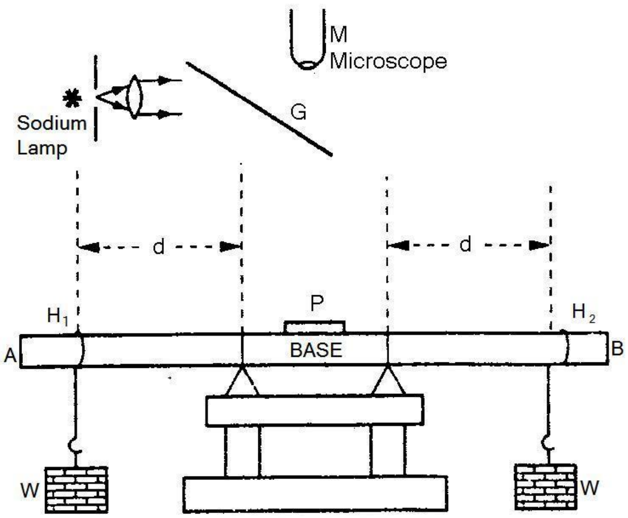

Fig. 3

Theory and Derivation of Formula: The experimental beam AB is bent under the action of the loads W, W hanging at both the ends. The bending moment G acting at all the transverse sections of the central span is related to the longitudinal radius of curvature R_l by

$$G = \frac{Y I}{R_l} \qquad (1)$$

Where Y is the Young's modulus for the material of the beam and I is the geometrical moment of inertia of cross section of the beam about an axis passing through its centroid and perpendicular to the plane of bending. If b is the breadth and t the thickness of the beam, then

$$I = \frac{1}{12} b\, t^3 \qquad (2)$$

If d is the distance between the point of support of the load W and knife edge nearer to it, then

$$G = W d \qquad (3)$$

From the eq. (1) and (3)

$$\frac{Y\ I}{R_l} = W\ d$$

If the beam has an initial radius of curvature R_0 due to its own weight, then

$$\frac{Y\ I}{R_l} - \frac{Y\ I}{R_0} = W\ d \tag{4}$$

If for a load W' the radius of curvature (longitudinal) of the beam is R_l', then in analogy to eq. (4), we have

$$\frac{Y\ I}{R_l'} - \frac{Y\ I}{R_0} = W'\ d \tag{5}$$

Subtracting eq (5) from (4), we have

$$Y\,I\left[\frac{1}{R_l} - \frac{1}{R_l'}\right] = (W - W')d$$

Therefore $$Y = \frac{(W - W')d}{I\left(\frac{1}{R_l} - \frac{1}{R_l'}\right)}$$

Substituting the value of I from eq.(2), we have

Formula 1st $$Y = \frac{12\,(W - W')d}{b t^3\left[\frac{1}{R_l} - \frac{1}{R_l'}\right]} \tag{6}$$

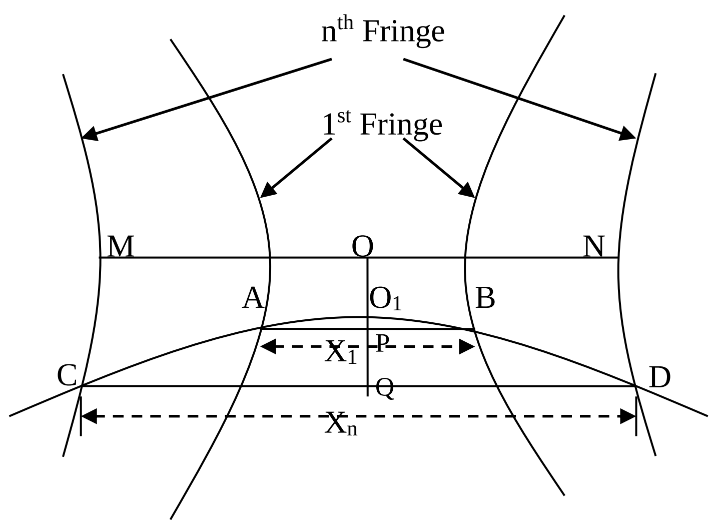

Fig. 4

In order to determine R_l and R_l' let us refer to Fig 4. Let MN be the lower surface of the cover plate and AB the curved section of the upper surface of the curved beam for weight W. Let O_l be the centre of the fringes system and A , B the position of the 1st pair of fringes in the longitudinal direction (along X-axis). Let $AB = X_1$ .From the geometry of the fig. 7, we have

$$(2\,R_l - O_l\,P)\,O_l\,P = \left(\frac{X_1}{2}\right)^2$$

But $O_lP << 2R_l$, hence neglecting $(O_lP)^2$ we have

$$2R_l\,(O_l\,P) = \left(\frac{X_1}{2}\right)^2$$

Similarly, if C and D represent the position of nth pair of fringes, separated by X_n, then

$$2R_l\,(O_lQ) = \left(\frac{X_n}{2}\right)^2$$

Subtracting theses two equations we have

$$2\,R_l\,(O_lQ - O_l\,P) = \tfrac{1}{4}(X^2_n - X^2_1)$$

But $(O_lQ - O_l\,P) = \frac{(n-1)\lambda}{2}$ for a bright fringe, where λ is the wavelength of Sodium light.

Formula 2nd

Therefore,

$$R_l = \frac{(X_n^2 - X_1^2)}{4\lambda(n-1)} \quad (7)$$

Similarly, if X_n' and X_1' corresponds to R_l' i.e. to weight W' then

Formula 3rd

$$R_l' = \frac{(X'^2_n - X'^2_1)}{4\lambda(n-1)} \quad (8)$$

Formula 1st is requisite formula for determining the value of Young's modulus Y for glass, the values of R_l and R_l' being given by **formula 2nd and 3rd** respectively.

Similarly, if Y_n and Y_1 referred to fringes in the transverse direction along Y-axis and R_l is the radius of anti-elastic curvature (as there will be contraction in the perpendicular direction of the beam) then

Formula 4th

$$R_t = \frac{(Y_n^2 - Y_1^2)}{4\lambda(n-1)} \quad (9)$$

Hence, the Poisson's ratio

$$\sigma = \frac{longitudinal\ radius\ of\ curvature}{transverse\ radius\ of\ curvature} = \frac{R_l}{R_t}$$

Formula 5th

$$\sigma = \frac{(X_n^2 - X_1^2)}{(Y_n^2 - Y_1^2)} \quad (10)$$

Procedure:

1. The experiment is arranged as shown in figure 1. Weights of certain mass, say 100 gm or 200 gm are placed on each hanger at both the ends of the experimental beam AB. The air film between the rectangular plate P and curved experimental beam is illuminated by means of an extended source of monochromatic light (sodium lamp), so that hyperbolic fringes are formed. The traveling microscope is focused on the fringes.

2. The distance between the 1st pair of fringes and the nth (say 4th) pair of fringes is determined in the longitudinal direction by means of microscope. These readings give X_1 and X_n respectively, similarly by measuring the distance in the transverse direction Y_1 and Y_n are determined. The Poisson's ration is then calculated from $\sigma = \frac{(X_n^2 - X_1^2)}{(Y_n^2 - Y_1^2)}$
3. Different sets of reading X_1, X_n and Y_1, Y_n are taken for different weights finally the mean value of σ is determined.
4. The values of R_l and R_l' are obtained by means of equation 7th and 8th, using the values of X_1, X_n, X_1', X_n' as measured above in two sets for weights W and W' (say 200 and 250 gms). Here n is 4 and λ for sodium light is 5893×10^{-8} cm.
5. The thickness t of the experimental beam is measured by screw gauge and the breadth b by vernier caliper. To measure the distance d, we measure the distance of the knife edges K_1 and K_2 from their respective end A and B (figure 6) and the average of two readings is taken.
6. On substituting the values of R_l and R_l' (as determined above), W = 200 gm, W' = 250 gm, the constants b, t and d in equation 6, the Young's modulus Y of the glass is calculated.

Observations:

(1) Distances X_1 and X_n measured in longitudinal direction, n = 4

Least Count of vernier of traveling microscope =…….cm.

S.No.	Weight W in gm.	No. of fringes	Distance (Diameter)			Distance X^2 cm^2
			Left end cm.	Right end cm.	Difference X cm.	
1st	200	1 n (say 4th)			X_1 =……. X_n =……	$(X_1)^2$ =…… $(X_n)^2$=……
					X'_1 =…… X'_n =……	$(X'_1)^2$ =….. $(X'_n)^2$ =…..
					X''_1 =….. X''_n =…..	$(X''_1)^2$ =…. $(X''_n)^2$ =….
	300	1 n (say 4th)			X'''_1 =…. X'''_n =….	$(X'''_1)^2$ =… $(X'''_n)^2$ =…

Calculations:

$$R_l = \frac{(X_n^2 - X_1^2)}{4\lambda(n-1)}$$

$$\frac{1}{R_l} = \frac{4\lambda(n-1)}{(X^2{}_n - X_1^2)} = \ldots\ldots\ldots\ldots \quad \text{cm}^{-1} \text{ for 150 gm}$$

Similarly

$$\frac{1}{R'_l} = \frac{4\lambda(n-1)}{(X'_n)^2 - (X'_1)^2} = \ldots\ldots \text{ cm}^{-1} \text{ for 200 gm}$$

$$\frac{1}{R''_l} = \frac{4\lambda(n-1)}{(X''_n)^2 - (X''_1)^2} = \ldots\ldots \text{cm}^{-1} \text{ for 250 gm}$$

$$\frac{1}{R'''_l} = \frac{4\lambda(n-1)}{(X'''_n)^2 - (X'''_1)^2} = \ldots \text{ cm}^{-1} \text{ for 300 gm}$$

(2) Distances Y_1 and Y_n measured in transverse direction, n = 4

S.No.	Weight W in gm.	No. of fringes	Distance (Diameter)			Distance Y^2 cm²
			Left end cm.	Right end cm.	Difference Y cm.	
1st	200	1 n (say 4th)			Y_1 =……. Y_n =……	$(Y_1)^2$ =…… $(Y_n)^2$=……
					Y'_1 =…… Y'_n =……	$(Y'_1)^2$ =….. $(Y'_n)^2$ =…..
					Y''_1 =….. Y''_n =…..	$(Y''_1)^2$ =…. $(Y''_n)^2$ =….
	300	1 n (say 4th)			Y'''_1 =…. Y'''_n =….	$(Y'''_1)^2$ =… $(Y'''_n)^2$ =…

Calculations:

$$R_l = \frac{(Y_n^2 - Y_1^2)}{4\lambda(n-1)}$$

$$\frac{1}{R_l} = \frac{4\lambda(n-1)}{(Y^2_n - Y_1^2)} = \ldots\ldots\ldots\ldots \text{ cm}^{-1} \text{ for 150 gm}$$

Similarly

$$\frac{1}{R'_l} = \frac{4\lambda(n-1)}{(Y'_n)^2 - (Y'_1)^2} = \ldots\ldots \text{ cm}^{-1} \text{ for 200 gm}$$

$$\frac{1}{R''_l} = \frac{4\lambda(n-1)}{(Y''_n)^2 - (Y''_1)^2} = \ldots\ldots \text{cm}^{-1} \text{ for 250 gm}$$

$$\frac{1}{R'''_l} = \frac{4\lambda(n-1)}{(Y'''_n)^2 - (Y'''_1)^2} = \ldots \text{ cm}^{-1} \text{ for 300 gm}$$

(3) Calculation of Poisson's Ratio σ :

S.No.	Poisson's Ratio σ	Mean σ
1st	$\sigma = \frac{R_l}{R_t} = \frac{X_n^2 - X_1^2}{Y_n^2 - Y_1^2} = \cdots$	
2nd	$\sigma = \frac{R_l'}{R_t'} = \frac{X'^2_n - X'^2_1}{Y'^2_n - Y'^2_1} = \cdots$	
3rd	$\sigma = \frac{R_l''}{R_t''} = \frac{X''^2_n - X''^2_1}{Y''^2_n - Y''^2_1} = \cdots$	

(4) Constant of the apparatus :

L.C. of screw gauge = ……cm.

L.C. of vernier callipers =……cm.

(i) Thickness t of the beam (by screw gauge) for W= 0.

S.No.	Zero error (cm)	Main scale reading (cm)	Vernier scale reading (cm).	Total reading (cm).	Mean t (cm).
1.					
2.					
3.					

(ii) Breadth b of the beam (By vernier callipers) for W= 0.

S.No.	Zero error (cm).	Main scale reading (cm).	Vernier scale reading (cm).	Total reading (cm).	Mean b (cm).
1.					
2.					
3.					

(iii) Distance d between knife edge and hanger for W=0.

S.No.	Distance $K_1 A_1$ (cm).	Distance $K_2 B_2$ (cm).	d= $\frac{K_1 A_1 + K_2 B_2}{2}$	Mean d cm.
1.				
2.				
3.				

NOTE:

Approx. dimension of experimental plate: Length = 50 cm. Breath 3.7 cm and Width = 1.85 mm.

Distance between the knife edge ; 23 cm.

'd' can be calculated by measuring the position of hanger

(5) Calculations of Young modulus:

λ for sodium light is 5893 x 10^{-8} cm.

Set 1st $$Y = \frac{12(W - W')d}{bt^3\left(\frac{1}{R_l} - \frac{1}{R_l'}\right)}$$, where W=150gm and W'=200 gm)

= …….x 10^{11} dynes/cm^2

Set 2nd $$Y = \frac{12(W'' - W''')d}{bt^3\left(\frac{1}{R''_l} - \frac{1}{R_l'''}\right)}$$, where W''=250gm and W'''=300gm)

= …….x 10^{11} dynes/cm^2

Similarly take other pairs e.g (W - W''), (W - W'''), (W' - W''), (W' - W''') and find the value of Y for each set.

Mean value of Y =………x10^{11} dynes/cm^2

Result: (i) Poisson's Ratio σ for glass = ……….

(ii)Young's Modulus Y for glass =…….. x 10^{11} dynes/cm^2

Standard Result: (i) Poisson's Ratio σ for glass = ……….

(ii)Young's Modulus Y for glass =……… x10^{11} dynes/cm^2

Sources of error and precautions:

1. The beam should be placed symmetrically on two knife edges.(i.e K_1 A and K_2 B must be equal).
2. Constants of the apparatus should be determined only when the beam is unloaded.
3. The experimental beam should not touch the base of traveling microscope.
4. All the glass plates should be optically plane and clean.
5. If the first few fringes are blurred then we take the reading for s (say 4th) and n + s (say 11); and Y, σ are determined by using the relations

$$R_l = \frac{(X_{n+s}^2 - X_s^2)}{4\lambda n} \text{ and } R_l = \frac{(Y_{n+s}^2 - Y_s^2)}{4\lambda n}$$

6. To eliminate effects arising due to imperfect flatness in the experimental beam and cover plate, the beam is turned over and the loads applied as usual. It is however necessary to place the cover plate below the beam and bring their surface together with the help of three leveling screws which are used to support the cover glass thus the mean value of X_n or Y_n is determined.