DIFFRACTION OF A SINGLE AND DOUBLE SLIT

APPARATUS:

He-Ne Laser, Slits, Detector, Digital micro meter, Screen, 1.5 meter long twin bar optical bench with suitable uprights.

PURPOSE OF THE EXPERIMENT:

To measure the intensity distribution due to single and double slits and to measure the slit width (d) and slit separation (a).

BASIC METHODOLOGY:

Light from a He-Ne Laser source is diffracted by single and double slits. The resulting intensity variation is measured by a photo cell whose outputs is read off a current measurement.

I INTRODUCTION:

1.1 Single Slit Diffraction:

We will study the Fraunhofer diffraction pattern produced by a slit of width *'a'*. A plane wave is assumed to fall normally on the slit and we wish to calculate the intensity distribution produced on the screen. We assumed that the slit consists of a large number of equally spaced point sources and that each point on the slit is a source of Huygen's secondary wavelets which interfere with the wane lets emanating from other secondary points. Let the point sources be at A_1, A_2, A_3, …and let the distance between the consecutive points be Δ. See (fig. 1). Thus, if the number point source be n, then

$$A = (n-1)\Delta \qquad \ldots(1)$$

We now calculate the resultant field produced by these *n* sources at point *P* on the screen. Since the slit actually consists of a continuous distribution of sources, we will in the final expression, let *n* go to infinity and Δ go to zero such that *n* Δ tends to *a*.

Now at point *P* the amplitudes the disturbances reaching from A_1, A_2, A_3, …will be very nearly the same because the point at a distance which is very large in comparison to *a*. However, because of even slightly different path lengths *ti* the point *P*, the field produced by A_1 will differ in phase from the field produced by A_2.

For an incident plane waves, the points A_1, A_2, A_3 are in phase and therefore the additional path travelling by the disturbance emanating from the point A_2, A_2'. This follows from the fact the optical paths A_1B_1, P and A_2', B_2, P are the same. If the diffracted rays make an angle θ with the normal to the slit the path difference would be

$$A_2,A_2', = \sin\theta \qquad \ldots(2)$$

The corresponding phase difference $\emptyset$ would be given by

$$\emptyset = \frac{2\pi}{\lambda}\Delta sin\theta \qquad \ldots(3)$$

Thus, if the field at the point P due to the disturbance emanating from the point A_1 is a $\cos(\omega t)$ then the field due to the disturbance emending from A_2 would be emanating a $\cos(\omega t - \emptyset)$. Now the difference in phase of the disturbance reaching from A_2 and A_3 will also be $\emptyset$ and thus the resultant field at the point P would be given by

$$E = E_0[\cos(\omega t) + \cos(\omega t - \emptyset) + \ldots\ldots + \cos(\omega t - (n-1)\emptyset)] \qquad \ldots(4)$$

Because

$$\cos(\omega t) + \cos(\omega t - \emptyset) + \ldots\ldots + \cos(\omega t - (n-1)\emptyset)$$

$$= \frac{\sin\frac{n\emptyset}{2}}{\sin\frac{\emptyset}{2}}\cos\left[\omega t - \frac{1}{2}(n-1)\emptyset\right] \qquad \ldots(5)$$

Thus

$$E = E_\theta cos\left[\omega t - \frac{1}{2}(n-1)\emptyset\right] \qquad \ldots(6)$$

Where the amplitude E_0 of the resultant field would be given by

$$E_\theta = \frac{E_\theta \sin\frac{n\emptyset}{2}}{\sin\frac{\emptyset}{2}} \qquad \ldots(7)$$

In the limit of n - > ∞ and Δ - > 0 in such a way that n Δ - > a we have

$$\frac{n\emptyset}{2} = \frac{n}{2}\frac{2\pi}{\lambda}\Delta\sin\theta -> \frac{\pi}{\lambda}a\sin\theta$$

Further

$$\emptyset = \frac{2\pi}{\lambda}\Delta\sin\theta = \frac{2\pi}{\lambda}\frac{a}{n}\sin\theta$$ would tend to zero and we may therefore

Write

$$E_\theta = \frac{E_\theta \sin\frac{n\emptyset}{2}}{\sin\frac{\emptyset}{2}} = nE_0\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi}{\lambda}a\sin\theta} = A\frac{\sin\beta}{\beta} \qquad \ldots(8)$$

Where

$$A = nE_0 \text{ and } \beta = \frac{\pi a \sin\theta}{\lambda} \qquad \ldots(9)$$

Thus

$$E = A\frac{sin\beta}{\beta}cos(\omega t - \beta) \qquad \ldots(10)$$

The corresponding intensity distribution is given by

$$I = I_0\frac{sin^2\beta}{\beta^2} \qquad \ldots(11)$$

Where I_0 represent the intensity at $\theta = 0$

1.2 POSITION OF THE MAXIMA AND MINIMA:

The variation of the intensity with β is shown in Fig. 2a. From eq.(11) it is obvious that

Intensity is zero when

$$\mathrm{B} = \mathrm{m}\,\pi, \mathrm{m} \neq 0 \qquad \ldots(12)$$

Or

$$\mathrm{a}\sin\theta = \mathrm{m}\,\lambda; \mathrm{m} = \pm 1, \pm 2, \pm 3 \text{(minima)}$$

In order to determine the position of the minima, we differentiate eq. (11) w.r.t β and set it equal to zero.

This gives

$$\mathrm{Tan}\ \beta = \beta \text{ (maxima)} \qquad \ldots(13)$$

The root β = 0 corresponds to the central maximum. The other roots can be found by determining the points of intersection of the curves y = β and y = tan β (Fig. 2b, c).

The intersections occur at β = 1.43 π, β = 2.46 π etc. and are known as the first second maximum etc. since $\left[\frac{\sin(1.43\pi)}{1.43\pi}\right]^2$ is about 0.0496, the intensity of the first maximum is about 4.96% of the central maxima. Similarly the intensities of the second and third maximum are about 1.885 and 0.83% of the central maximum respectively.

1.3 DOUBLE SLIT DIFFRACTION PATTERN:

In this section we will study the Fraunhofer diffraction pattern produced by two parallel slits (each of width a) separates by a distance d. We would find that the resultant intensity distribution is a product of single slit diffraction pattern and the interference pattern produced by two point sources separated by a distance d.

In order to calculate the diffraction, we use a method similar to that used for the case of a single slit and assume that the slits consists of a large number of equally spaced point sources and that each point on the slit is a source of Huygen's secondary wavelets. Let the point sources be at

A_1, A_2, A_3…..(in the first slit) and at b_1, b_2, b_3…..(in the second slit) (see Fig. 3). As before, we assume that the distances between two consecutive points in either of the slits is Δ. Then the path difference between the disturbance reaching the point *P* from two consecutive point in a slit will be $\Delta \sin\theta$. The field produced by the first slit at the point *P* will, therefore be given by (see eq.)

$$E_1 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta) \qquad \ldots(14)$$

Similarly, the secondary slit will produce a field

$$E_2 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \Phi_1) \qquad \ldots(15)$$

At the point P where

$\Phi_1 = \frac{2\pi}{\lambda} d\sin\theta$ Represents the phase difference between the disturbance from two corresponding points on the slits by corresponding points we supply pair of points like (A_1, B_1), (A_2, B_2)…..which are separated by a distance d.

Hence the resultant field will be

$$E = E_1 + E_2 = A\frac{\sin\beta}{\beta}[\cos(\omega t - \beta) + \cos(\omega t - \beta - \Phi_1)]$$

Which represents the interference of two waves each of amplitude $A\frac{\sin\beta}{\beta}$ and differing in phase by Φ_1. Above equation can be written as

$$E - A\frac{\sin\beta}{\beta}\cos\gamma\cos\left(\omega t - \beta - \frac{\Phi_1}{2}\right)$$

Where

$$\gamma = \frac{\Phi_1}{2} = \frac{\pi}{\lambda} d\sin\theta$$

The intensity distribution will be of the form

$$I = 4I_0\frac{\sin^2\beta}{\beta^2}\cos^2\gamma \qquad \ldots(16)$$

Where $I_0 \frac{\sin^2\beta}{\beta^2}$ represents the intensity distribution produced by one of the slits. As a can be seen, the intensity distribution is a product of two terms, the first term $\frac{\sin^2\beta}{\beta^2}$ represent the diffraction produced by a single slit of width *a* and the second term ($cos^2\gamma$) represents the interference pattern produced by two points sources separated by a distance *d* (see Fig. 4).

1.4 POSITION OF MAXIMA AND MINIMA

Equation (16) tell us that the intensity is zero wherever $\beta = \pi, 2\pi, 3\pi.....$

Or when $\gamma = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, ...$

The corresponding angles of diffraction will be given by

$$a \sin\theta = m\lambda;\ (m = 1, 2, 3...)$$

and

$$d \sin\theta = \left[n + \frac{1}{2}\right]\lambda\ (n = 0, 1, 2, 3.....) \qquad ...(17)$$

Interference maxima occur when

$$\gamma = 0, \pi, 2\pi, 3\pi.....$$

Or when

$$d \sin\theta = 0, \lambda, 2\lambda, 3\lambda.....$$

II. SET-UP AND PROCEDURE:

1. Switch on the laser source about 15 minutes before the experiment is due to start. This ensures that the intensity of light from the laser source is constant.
2. Allow the laser beam to fall on a slit provided. Align the laser source and slit to get the clear diffraction pattern obtained on the screen.
3. The intensity distribution in the diffraction pattern is scanned with the help of a photodetector. The detector is secured to a mount and is kept as far behind the slit as possible. A screen with a slit (0.03mm wide) is fitted in front of the photodetector. The current is measured with Digital Microammeter which is approximately proportional to intensity of the incident light.
4. Repeat the same procedure for double slit and record the diffraction pattern on both the sides of central maximum. The interval between two consecutive minima positions of the detector should be small enough, so that the adjacent maxima/minima of the intensity distribution are missed.

PRECAUTIONS:

1. The laser beam should not penetrate into eyes as this may damages the eyes permanently.
2. The detector should be as away from the slit as possible.
3. The laser should be operated at a constant voltage 220V obtained from a stabilizer. This avoids the flickering of the laser beam.

EXPERIMENT NO.1

OBJECT:

To study the Single slit diffraction and Determine the width of single slit.

APPARATUS USED:

Helium Neon Laser source or diode laser with power supply, Single slit, Detector, Digital Microammeter, Screen, an optical bench 1.5 meter long with suitable uprights to mount uprights to slit, detector and laser.

FORMULA USED:

The width ‘d’ of the slit is

$$d = \frac{2D\lambda}{\beta}$$

Where

‘d’ = Width of single slit

β = Width of central maxima. The distance between the first order minima on both side of central maxima.

D = Distance between the slit and detector or screen.

EXPERIMENT NO.2

OBJECT:

To study the Double Slit diffraction and Determine the width of double slit.

APPARATUS USED:

Helium Neon Laser source or diode laser with power supply, Double slit, Detector, Digital Microammeter, Screen, An Optical bench 1.5 meter long with suitable uprights to mount uprights to slit, detector and laser.

FORMULA USED:

The width ‘d’ of the slit is

$$d = \frac{\lambda D}{\beta}$$

Where

‘d’ = Width of single slit

β = The distance from the centre of the first bright bar to the centre of third bright bar side of central maxima.

D = Distance between the slit and detector or screen

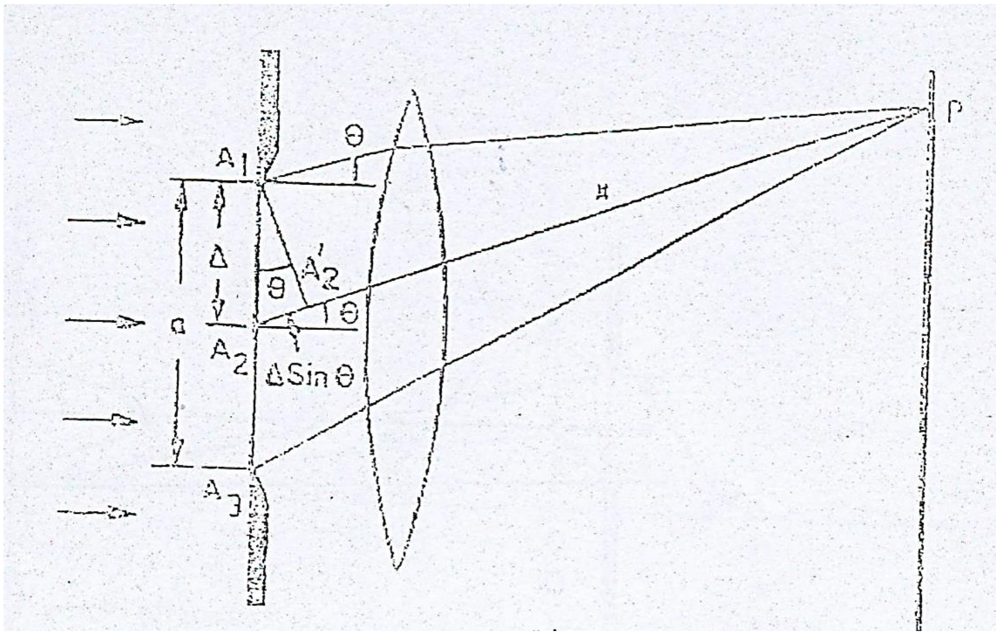
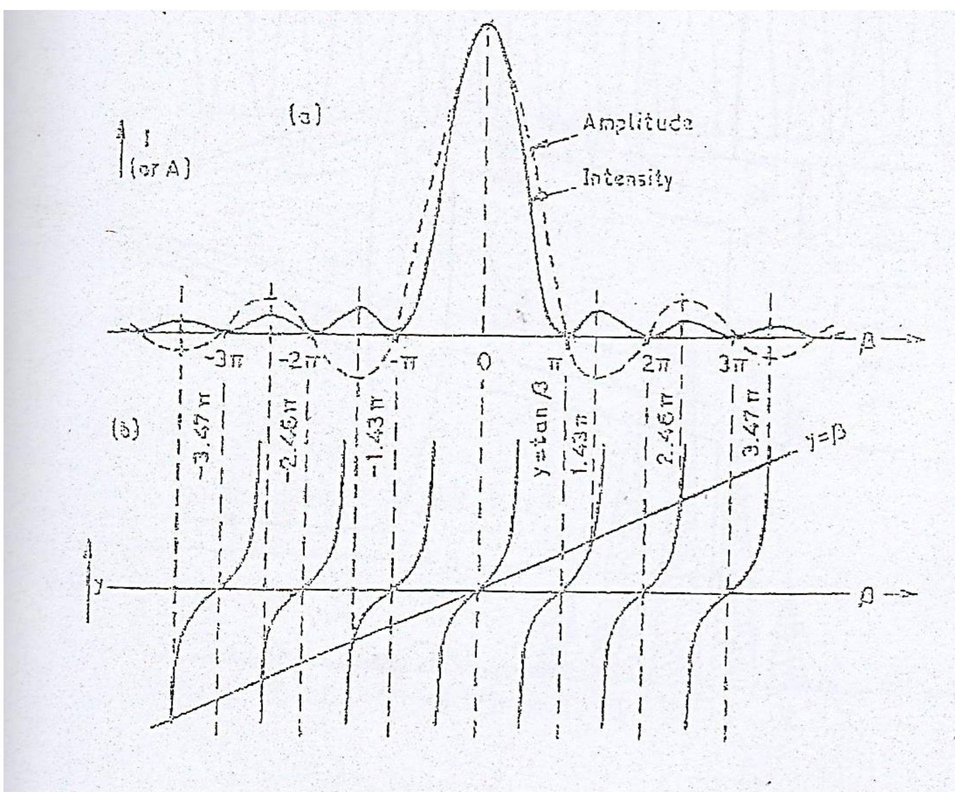

P
θ
A_1
Δ
A_2'
θ
θ
a
A_2
ΔSin θ
A_3

Fig. 1


(a)
I
(or A)
Amplitude
Intensity
β
−3π
−2π
−π
0
π
2π
3π
(b)
−3.47π
−2.46π
−1.43π
y = tan β
1.43π
2.46π
3.47π
y = β
y
β

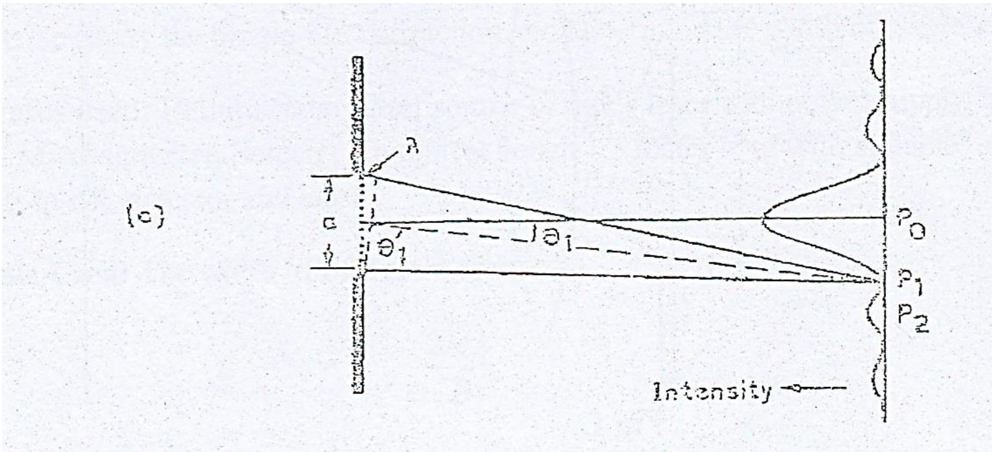

(c)
A
a
θ'1
θ1
P0
P1
P2
Intensity

Fig. 2

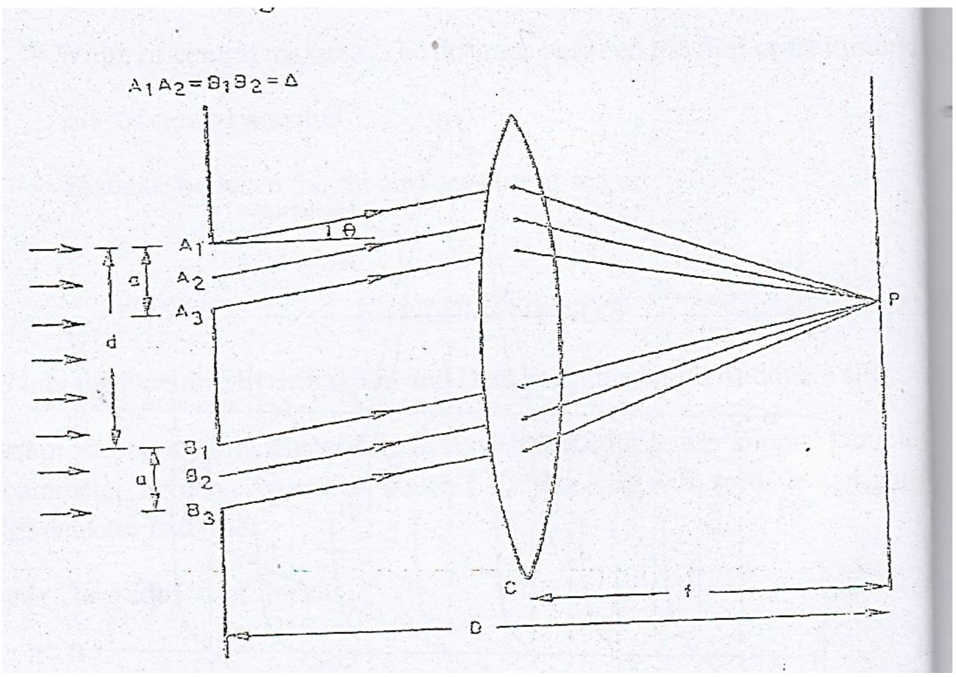
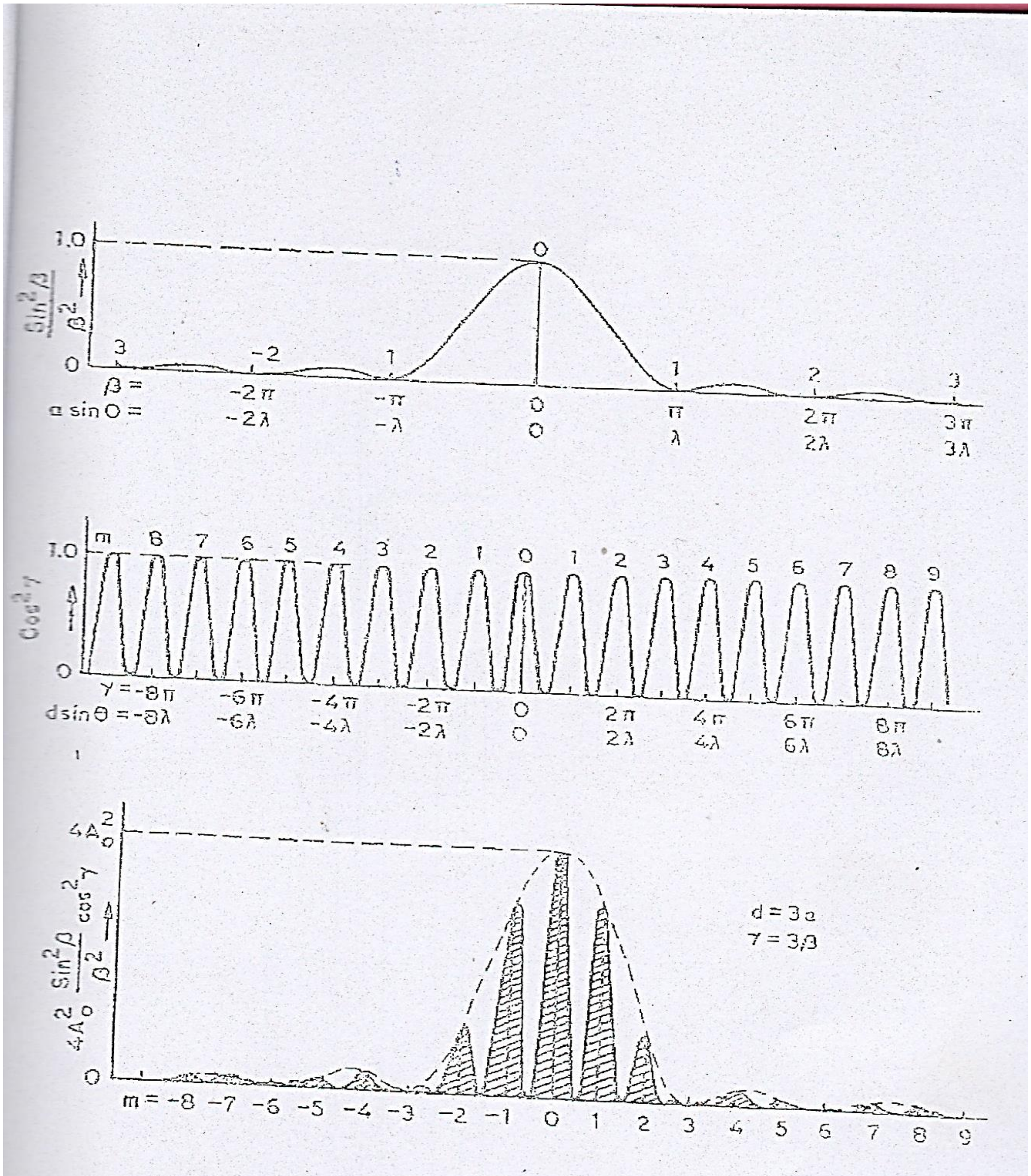

A1A2=B1B2=Δ
A1
A2
A3
a
d
θ
B1
B2
B3
a
P
C
f
D

Fig. 3


d = 3a
γ = 3β
β =
a sin θ =
γ = -8π
d sin θ = -8λ
m = -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9

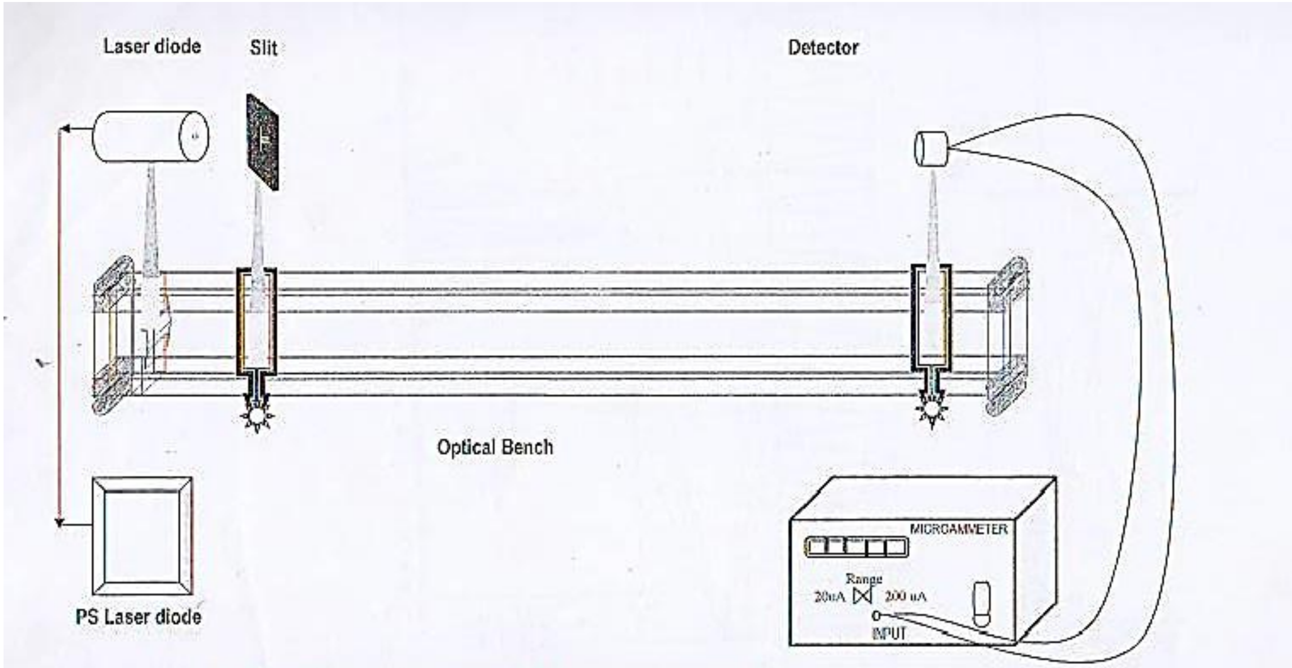


Fig.5 SET-UP TO STUDY THE DIFFRACTION OF LASER USING SLITS