Experiment No. 16

Viscosity

Aim:

To verify Stoke's law and hence to determine the coefficient of viscosity of a highly viscous liquid.

Apparatus and Accessories:

A glass cylinder of about 5 cm (or more) in diameter and about 100 cm in length**,** few spherical metal balls of diameters ranging from 1 to 3 mm, stop watch, screw gauge, meter scale, experimental liquid.

Formula Used :

Coeffficent of viscosity of experimental liquid (glycerine) is given by

$$\eta = \frac{2}{9}\frac{(\rho - \sigma)gr^2}{v}$$

σ = density of glycerine.
ρ = density of material of ball.
r = radius of spherical ball.
g = acceleration to gravity.
ν = terminal velocity.

Where, η is the coefficient of viscosity of fluid, ρ and σ represent respectively the density of the material of the ball and that of the fluid, and g is the acceleration due to gravity and is the radius of the ball falling in the experimental liquid.

Theory:

When a spherical ball of (radius r) is dropped in a viscous field it moves in it with certain velocity 'ν' (say) it experiences an opposing force(viscous force F_d). According to Stoke's law this viscous force is given by

$$F_d = 6\pi\eta r v$$

Simultaneously it experiences an upthrust (or buoyant force) F_b and gravitational force F_g. F_g tries to increase the velocity of ball whereas F_d decreases the velocity. After some time the ball will move with a steady velocity, called the terminal velocity. Under the steady condition.

$$F_g = F_b + F_d$$

Or $$F_d = F_g - F_b$$

$$6\pi\eta r v = \frac{4}{3}\pi r^3(\rho - \sigma)g$$

Or, $$\eta = \frac{2}{9}\frac{(\rho - \sigma)gr^2}{v}, \qquad (1)$$

where ρ and σ represent respectively the density of the material of the ball and that of the fluid, and g is the acceleration due to gravity.

In actual practice the experiment is performed in a liquid column of finite depth H contained in a cylinder of inner radius R. To take into account the effect of the finite depth and radius of the liquid column two corrections, known as Ladenburg corrections, are introduced as multipliers of the observed velocity 'v'. Thus the terminal velocity v is given by

$$v = v'(1 + \frac{2.4r}{R})(1 + 3.3\frac{r}{H}) \ , \qquad (2)$$

where the first correction term accounts for the finite radius of the liquid column and the second correction term is used for the finite depth of the liquid column. The relations (1) and (2) are the working formulae of the experiment.

In SI units, the radii r and R are expressed in m, H in m, v' in m/s, the densities ρ and σ in kg/m^3. And g in m/s^s. Then h is obtained in N.s/m^2 or Poiseulle (PI).

The relation (1) is deduced from Stoke's law and indicates that for a given liquid at a given temperature the ratio r^2/v should be a constant. Thus, the verification of Stoke's law requires that a graph of r^2 along the x-axis and 'v' along the y-axis should be a straight line. By using a value of r^2/v from this graph and measuring all other physical parameters appearing on the right-hand side of Eq.(1), the coefficient of viscosity η of the liquid can be determined.

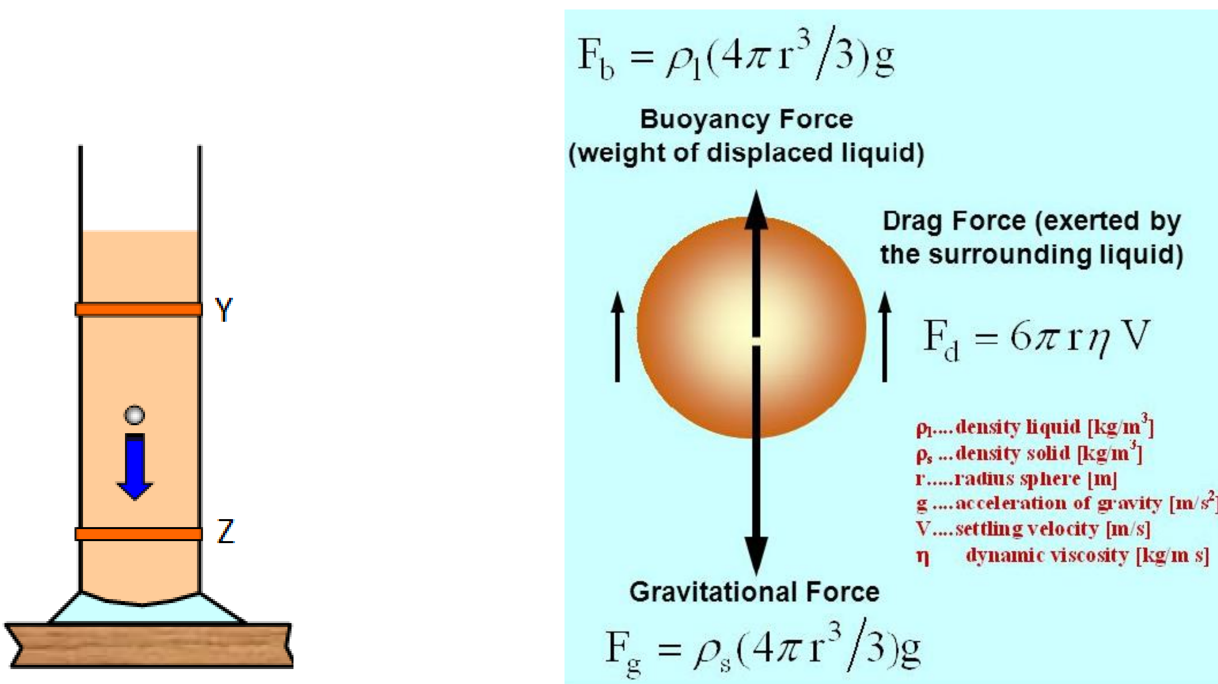

Fig. 1 The glass cylinder with the liquid

Procedure:

1. Measure the inner diameter of the cylinder at various places by slide calipers. Fin the mean diameter (2R) and hence the radius (R) of the cylinder.

2. Select three sets of balls; measure the diameter (2r) and hence the radius (r) of each ball by screw gauge. If the balls are very small, use a microscope for the measurement of the radius.

3. Set the cylinder vertically on a stand and pour the experimental liquid slowly. Measure the height H of the liquid column by a meter scale. Put horizontal marks Y and Z on the outer surface of the cylinder.

4. Wet the balls thoroughly in the experimental liquid and then drop one ball from each set gently one by one starting from the largest size with the help of a spatula into the liquid in the cylinder so that they fall centrally. For each ball, note the times of crossing of the distances Y and Z by a stop watch. Note the reading in Table 2.

5. After adjusting the positions of the markings Y and Z, drop the balls of one set gently again one by one and note by a stop watch the time taken by each ball in crossing the marks Y and Z. Measure the distance YZ by a meter scale and obtain the terminal velocity (v') by dividing the distance by the mean time for a set of balls. Repeat this procedure for the other sets.

6. Calculate v from v', r, R and H. Find the value of r^2/v for each set of balls. The constancy of this value for all the sets proves the validity of Stoke's law.
7. Also draw a graph by putting r^2(in m^2) along the x-axis and v (in m/s) along the y-axis. This will be a straight line passing through the origin. This nature of the graph also proves the validity of Stoke's law.
8. Find the value of r^2/v corresponding to a point on the graph sufficiently removed from the origin and calculate the viscosity η of the liquid using Eq. (1).

Observations:

Inner radius of Cylinder (R) = 2.5 cm
Distance between Y and Z (H) = 60 cm
Density of material of balls (ρ) = 7.8 gm/cm^3
Density of liquid (σ) = 1.26 gm/ cm^3

Table 1: For radii 'r' of the balls

Least count of screw gauge ……..

No. of sets	Diameter (2r) of the ball (cm)					Mean r (cm)
	Ball numbers	m.s. (cm)	Vernier scale (v.d. × v.c.)	Total (cm)	Mean (cm)	
1	1 2 3 etc.	… … …	… … …	… … …	…	…
2					…	…
etc.					…	…

Table 2: Measurement of v' and determination of v

Length of the middle portion of the liquid column (YZ), l = … cm

No. of sets	Ball numbers	Mean radius, r (cm) from Table 1	Time of fall (sec.)	Mean time, t (sec.)	Observed velocity v' (= l/t) (cm/sec.)	Corrected $v = v'(1+2.4\frac{r}{R})\times(1+3.3\frac{r}{H})(cm/\sec.)$
1	1 2 3 etc.	…	… … … etc.	…	…	….
2	1 2 3 etc.	…	… … … etc.	…	…	….
etc.	1 2 3 etc.	…	… … … etc.	…	…	….

Table 3:

Verification of Stoke's law and determination of η

g= …m/s^2

No. of sets	Mean radius r (cm)	Value of r^2 (m^2)	Value of v from Table 2 (m/s)	r^2/v (m.s)	r^2/v from graph (m.s)	$\eta = \frac{2}{9}(\rho-\sigma)g(r^2/v)(N.s/m^2)$
1	…	…	…	…	…	….
2	…	…	…	…		
3	…	…	…	…		

Result:

1. The graph of r^2 along the x-axis and v along the y-axis is found to be a straight line hence Stoke's law is verified.
2. Viscosity of experimental liquid (glycerine) is …..

Computation and percentage error:

We have from eqs. (1) and (2)

$$\eta = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{v'K},$$

Where K = $(1+2.4\frac{r}{R})(1+3.3\frac{r}{H})$

Or, $$\eta = \frac{1}{18}.\frac{D^2}{l}.\frac{(\rho-\sigma)gt}{K},$$

Since $v' = \frac{l}{t}$ and $r = \frac{D}{2}$, where D is the diameter of a ball.

Hence the proportional error in η is

$$\frac{\delta\eta}{\eta} = \frac{2\delta D}{D'} + \frac{\delta(\rho-\sigma)}{\rho-\sigma} + \frac{\delta t}{t} + \frac{\delta l}{l} + \frac{\delta K}{K}.$$

Since the values of l, R and H are fairly large and the values of ρ and σ can be determined fairly accurately, the contributions of $\frac{\delta(\rho-\sigma)}{\rho-\sigma}$, $\frac{\delta l}{l}$ and $\frac{\delta K}{K}$ to the total proportional error in η are very small and can be neglected. Therefore the percentage error in η is given by

$$\frac{\delta\eta}{\eta}\times 100 = 2\frac{\delta D}{D'}\times 100 + \frac{\delta t}{t}\times 100 .$$

This indicates that maximum care should be taken to measure D, i.e., 2r and t in the measurement of η. Use the least count of the screw gauge and the value of the smallest division of the stop watch to calculate $\frac{\delta\eta}{\eta}\times 100$.

Precautions:

1. The radii of the spheres must be measured very accurately since r occurs in the expression of η in the second power. When the balls are very small, a microscope should be used to measure their radii.

2. Before the balls are dropped into the liquid of the cylinder ensure that they are wetted thoroughly in the experimental liquid; otherwise a layer of air surrounding each ball will affect the result.

3. Since the viscosity changes rapidly with the temperature of the liquid, care should be taken to maintain the temperature of the liquid constant during the experiment.

4. As an error in the measurement of t contributes much to the proportional error in η, care should be taken to measure it as accurately as possible.

5. When the size and the number of the balls are small, determine ρ by dividing mass by volume of the balls; otherwise, a large error in the measurement of ρ by the specific gravity bottle will occur.