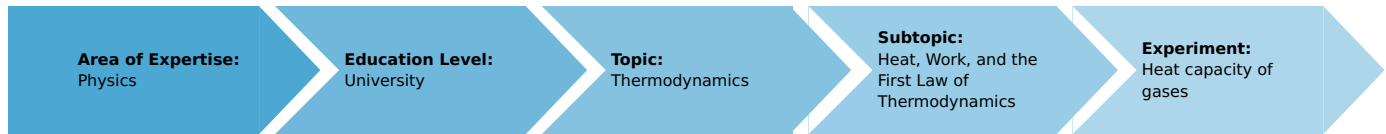


# Heat capacity of gases (Item No.: P2320201)

## Curricular Relevance



### Difficulty



Intermediate

### Preparation Time



1 Hour

### Execution Time



3 Hours

### Recommended Group Size



2 Students

### Additional Requirements:

### Experiment Variations:

- Datalogging: P2320261 Heat capacity of gases with Cobra4

### Keywords:

Equation of state for ideal gases, 1st law of thermodynamics, Universal gas constant, degree of freedom, Mole volumes, Isobars, Isotherms, Isochores, Adiabatic changes of state

## Overview

### Short description

#### Principle

Heat is added to a gas in a glass vessel by an electric heater which is switched on briefly. The temperature increase results in a pressure increase, which is measured with a manometer. Under isobaric conditions a temperature increase results in a volume dilatation, which can be read from a syringe. The molar heat capacities  $C_V$  and  $C_p$  are calculated from the pressure and volume change respectively.

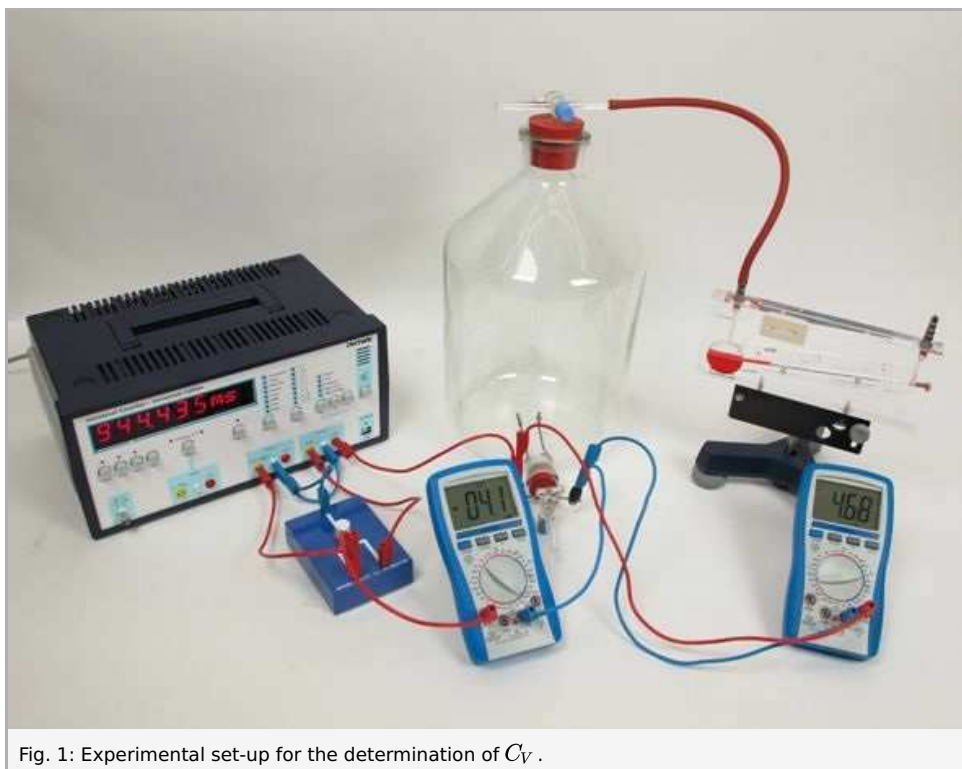


Fig. 1: Experimental set-up for the determination of  $C_V$ .

## Equipment

Position No.	Material	Order No.	Quantity
1	Universal counter	13601-99	1
2	Stopcock,1-way,straight, glass	36705-00	1
3	Stopcock,3-way,t-sh.,capil.,glass	36732-00	1
4	Rubber stopper 26/32, 3 holes 7 mm + 2 x 1,5 mm	39258-14	1
5	Rub.stop.d=59.5/50.5mm, 1 hole	39268-01	1
6	Rubber tubing, i.d. 6 mm	39282-00	2
7	Silicone tubing, inner diameter 3 mm	39292-00	1
8	Nickel electrode,d 3mm,w.socket	45231-00	2
9	Tubing adaptor, ID 3-5/6-10 mm	47517-01	1
10	Scissors,straight,blunt,l 140mm	64625-00	1
11	Weather monitor, 6 lines LCD	87997-10	1
12	Tripod base PHYWE	02002-55	1
13	Syringe 10ml, Luer, 10 pcs	02590-03	1
14	Mariotte flask, 10 l	02629-00	1
15	Precision manometer	03091-00	1
16	Two-way switch, single pole	06030-00	1
17	Chrome-nickel wire, d.0,1mm,100m	06109-00	1
18	Digital multimeter 2005	07129-00	2
19	Connecting cord, 32 A, 100 mm, blue	07359-04	2
20	Connecting cord, 32 A, 250 mm, red	07360-01	1
21	Connecting cord, 32 A, 250 mm, blue	07360-04	1
22	Connecting cord, 32 A, 500 mm, red	07361-01	4
23	Connecting cord, 32 A, 500 mm, blue	07361-04	1

## Tasks

1. Determine the molar heat capacities of air at constant volume  $C_V$ .
2. Determine the molar heat capacities of air at constant pressure  $C_p$ .

## Set-up and procedure

### Set-up

- Perform the experimental set-up according to Figs. 1 and 2 respectively.

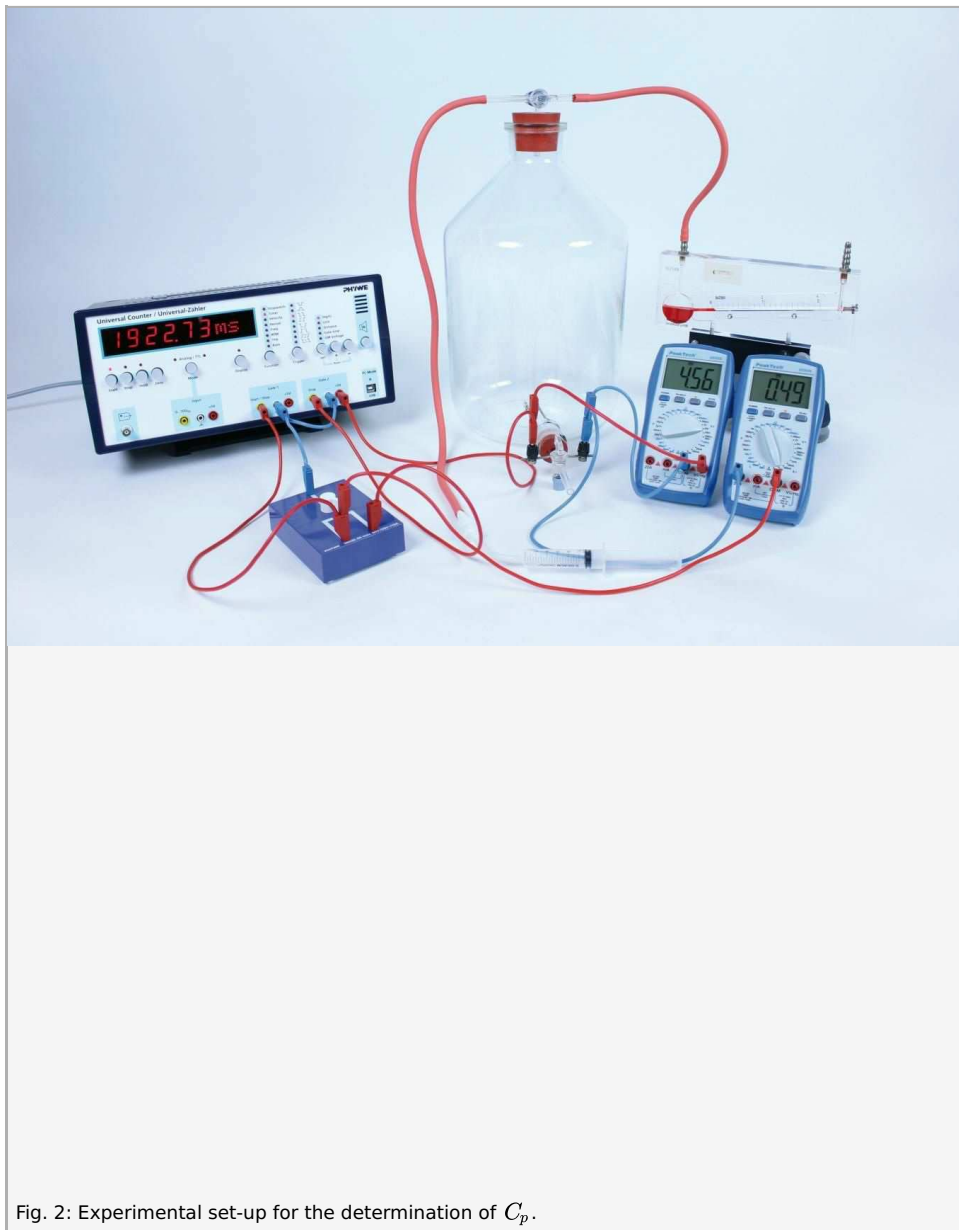


Fig. 2: Experimental set-up for the determination of  $C_p$ .

- Insert the two nickel electrodes into two holes of the three hole rubber stopper and fix the terminal screws to the lower ends of the electrodes.
- Screw two pieces of chrome nickel wire, which are each about 15 cm long, into the clamps between these two electrodes so that they are electrically connected in parallel. The wires must not touch each other.
- Insert the one-way stopcock into the third hole of the stopper and insert the thus prepared stopper in the lower opening of the bottle. Give special attention to the wires which have to protrude into the middle of the bottle.
- Insert the second stopper, which has been equipped with the three-way stopcock, into the upper opening of the bottle (Fig. 1) and connect the precision manometer to the bottle with a piece of tubing.
- The manometer must be positioned exactly horizontally.
- It is equipped with a spirit level to facilitate the correct adjustment. Use the adjusting screws of the tripod base to align the manometer completely horizontally.
- The manometer must be filled with the oil which is supplied with the device.
- The scale is now calibrated in hPa.
- You can choose the scale of either 2 hPa or 4 hPa by altering the inclination angle of the manometer. For these measurements 2 hPa are sufficient so just leave it horizontal.
- One of the 5 V outputs of the universal counter serves as the power source. The electrical circuit is illustrated in Fig. 3 and Fig. 4.

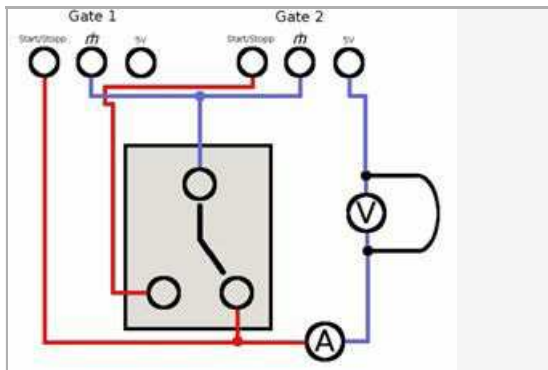


Fig. 3: Schematic circuit for measuring the heating time.

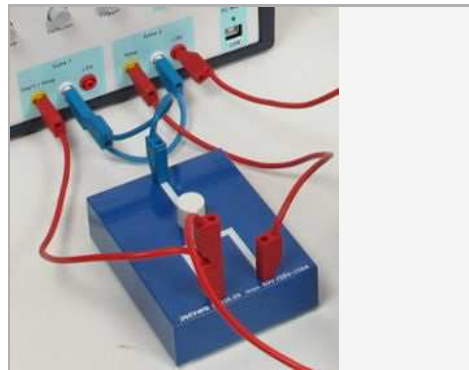


Fig. 4: Connecting two-way switch and counter.

- To determine  $C_p$  connect the syringe to the bottle via the three-way stopcock (compare Fig. 2).

## Procedure

- For each task perform at least ten measurements.
- The rise tube of the manometer must be well wetted before each measurement.
- As the counter has to measure the heating time choose the following settings:  
Function: Timer  
Trigger:  $\square\square$
- Determine the current which flows through the heating wire and the voltage separately at the end of the measuring series. To achieve this, connect one of the digital multimeters in series as an ammeter and the other in parallel as a voltmeter (compare Fig. 3). Determine the air pressure, which is required for the calculations, with the aid of the weather station.

### Task 1:

- Start and stop the measuring procedure by operating the two-way switch.
- The measuring procedure should be as short as possible (less than two seconds).
- The three-way cock must be positioned in such a manner, that it connects the bottle with the precision manometer.
- Upon heating the pressure in the bottle will start to rise.
- Read the maximum pressure increase immediately after cessation of the heating process.
- After each measurement wait a sufficient time until the gas in the volume cooled down again to room temperature thereby regaining ambient pressure.
- The electrical current which flows during the measurements must not be too strong, i.e. it must be sufficiently weak to limit the pressure increase due to the heating of the gas to a maximum of 1 hPa.
- For this reason it may be necessary to use only one heating wire or to reduce the electrical current at the power supply.

### Task 2:

- Start and stop the measuring procedure by operating the two-way switch.
- The measuring procedure should be as short as possible (less than two seconds).
- While measuring, the three-way cock must be positioned in such a manner that it connects the syringe and the manometer with the bottle.
- Upon heating the pressure in the bottle will start to rise.
- As you want to determine the heat capacity at constant pressure you have to compensate the pressure rise by increasing the volume via the syringe.
- You can hold the syringe in your hand and use your thumb to gently push the plunger.
- When the heating stopped, the volume of the gas in the bottle will still increase for a moment.
- Be careful to notice the turning point when the volume starts decreasing again because the gas starts cooling down. In this moment the pressure should have its initial value and start falling while you have already stopped increasing the volume.
- You can read the volume increase directly from the syringe's scale. You may need some practice until you are able to keep the pressure fairly constant during the whole measurement and recognize the turning point correctly.
- After each measurement reset the initial volume and wait until the gas cooled down again to room temperature.

- Before starting a new measurement both the volume in the syringe and the pressure should have regained their initial values.



Fig. 5: For the second task operate the syringe with one hand while operating the switch with the other hand.

## Theory and evaluation

### Theory

### Evaluation

In the following the evaluation of the obtained values is described with the help of example values. Your results may vary from those presented here.

*Task 1: Determine the molar heat capacities of air at constant volume  $C_V$ .*

Under isochoric conditions, the temperature increase  $dT$  produces a pressure increase  $dp$ . The pressure measurement results in a minute alteration of the volume which must be taken into consideration in the calculation:

$$dT = \frac{p}{nR}dV + \frac{V}{nR}dp = \frac{T}{pV}(pdV + Vdp) \quad (11)$$

It follows from equations (3) and (1) that:

$$C_V = \frac{1}{n} \cdot \frac{dQ - pdV}{dt} \quad (12)$$

The energy  $dQ$  is supplied to the gas by the electrical heater:

$$dQ = U \cdot I \cdot dt \quad (13)$$

There  $U$  is the voltage which is applied to the heater wires,  $I$  is the current which flows through the heater wires and  $dt$  is the period of time of the measurement.

With equations (11) and (13) one obtains:

$$C = \frac{p \cdot V}{n \cdot T} \cdot \frac{U \cdot I \cdot dt - p \cdot dV}{p \cdot dV + V \cdot dp} \quad (14)$$

where  $dV$  is the volume change due to the rising oil in the manometer.

The indicator tube in the manometer has a radius of  $r = 2 \text{ mm}$  and a length of  $l = 140 \text{ mm}$ . The pressure change per length is accordingly  $1/70 \text{ hPa} \cdot \text{mm}^{-1}$  and the corresponding change in volume is therefore:

$$dV = a \cdot dp \tag{15}$$

where

$$a = \pi r^2 \cdot 70 \frac{\text{mm}}{\text{hPa}} = 880 \frac{\text{mm}^3}{\text{hPa}} = 8.8 \cdot 10^{-4} \frac{\text{l}}{\text{hPa}} \tag{16}$$

thus

$$C_V = \frac{pV \cdot (dQ - a \cdot p \cdot dp)}{n \cdot T \cdot (a \cdot p + V) \cdot dp} \tag{17}$$

The molar volume of a gas at standard pressure  $p_0 = 1013 \text{ hPa}$  and  $T_0 = 273.2 \text{ K}$  is  $V_0 = 22.4141/\text{mol}^{-1}$ .

The molar volume is:

$$V_{\text{mol}} = \frac{p_0 \cdot V_0 \cdot T}{T_0 \cdot p} \tag{18}$$

In accordance with the following, the number of moles in volume  $V$  is:

$$n = \frac{V}{V_{\text{mol}}} \tag{19}$$

Taking equations (18) and (19) into consideration, it follows that:

$$C_V = \frac{p_0 \cdot V_0}{T_0} \cdot \left( \frac{U \cdot I \cdot dt}{(ap + V) \cdot dp} - \frac{ap}{ap + V} \right) \tag{20}$$

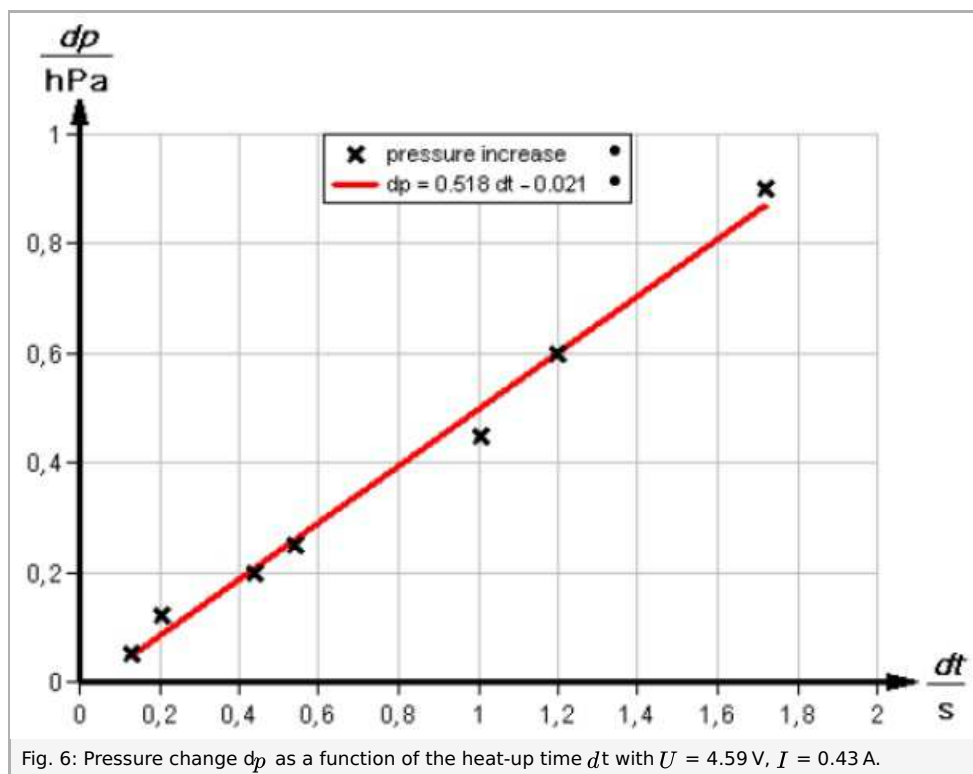


Fig. 6: Pressure change  $dp$  as a function of the heat-up time  $dt$  with  $U = 4.59 \text{ V}$ ,  $I = 0.43 \text{ A}$ .

The slope of the linear regression in Fig. 6 is equal to:

$$\frac{dp}{dt} = 0.518 \frac{\text{hPa}}{\text{s}}$$

$C_V$  can be calculated using equation (20) if equation (16) is taken into consideration.

With

$$p = 1011 \text{ hPa}$$

$$V = 101$$

$$U = 4.59 \text{ V}$$

$$I = 0.43 \text{ A}$$

The following value for  $C_V$  is obtained:

$$C_V = 21.67 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1} \pm 5 \% \quad (21)$$

*Task 2: Determine the molar heat capacities of air at constant pressure  $C_p$ .*

At constant pressure the temperature increase  $dT$  induces a volume increase  $dV$ . From the equation of state for ideal gases follows that:

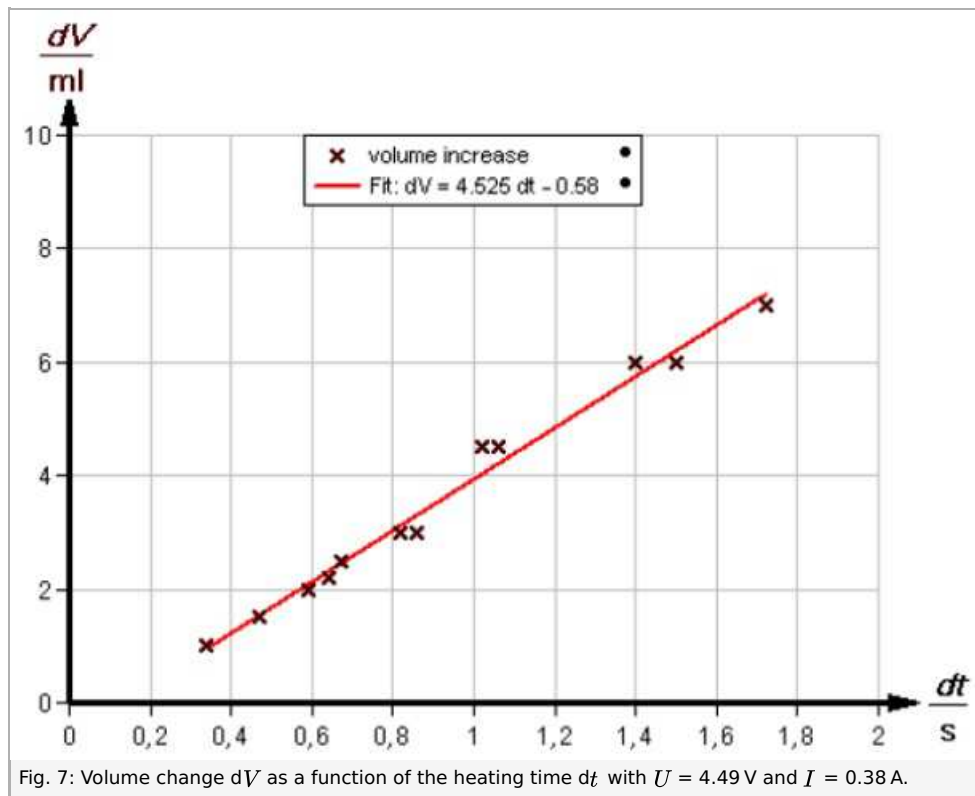
$$dV = \frac{nR}{p} dt = \frac{V}{T} dT \quad (22)$$

Taking equation (2) into consideration, the following results from equations (13) and (22):

$$C_p = \frac{1}{n} \cdot \frac{U \cdot I \cdot dt \cdot V}{dV \cdot T} \quad (23)$$

$C_p$  can be calculated using equation (23) under consideration of (18) and (19):

$$C_p = \frac{p_0 \cdot V_0}{T_0} \cdot \left( \frac{UI}{p} \right) \cdot \left( \frac{dt}{dV} \right) \quad (24)$$



The slope of the linear regression in Fig. 7 is equal to

$$\frac{dV}{dT} = 4.53 \frac{\text{ml}}{\text{s}}$$

with

$$U = 4.49 \text{ V and } I = 0.38 \text{ A}$$

From which follows

$$C_p = 30.98 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1} \pm 7\% \tag{25}$$

As a consequence of heat losses to the surroundings the experimental values for  $C_p$  and  $C_V$  are somewhat larger than the theoretical values.

The difference between the molar heat capacities provides the value for  $R$ . The experimental results give

$$R = C_p - C_V = 9.31 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1} \pm 9\%$$

Which is congruent to the value given in the literature of  $R = 8.3 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$ .

**Literature values:**



$$C_{p(\text{Oxygen})} = 29.4 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$$

$$C_{V(\text{Oxygen})} = 21.1 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$$

$$C_{p(\text{Nitrogen})} = 29.1 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$$

$$C_{V(\text{Nitrogen})} = 20.8 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$$

$$R = 8.314 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$$

**Experimental results:**

$$C_{p(\text{air})} = 31 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$$

$$C_{V(\text{air})} = 22 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$$

**Note**

Using this apparatus, other gases (e.g. carbon dioxide or argon) can also be measured. These gases are then introduced through the stopcock on the bottom of the vessel.