Indian Institute of Technology Roorkee

MAI-101: Mathematics I Autumn Semester 2024–2025

Assignment 8: (Applications of Multiple Integrals)

1. Find the area of the region included between the two cardioid $r = a(1 + cos\theta)$ and $r = a(1 - cos\theta)$.

2. Find the area of that part of the cylinder $x^2 + y^2 = a^2$ which is cut out by the cylinder $x^2 + z^2 = a^2$.

3. Find the volume of a wedge intercepted between the cylinder $x^2 + y^2 = 2ax$ and the planes $z = mx$ and $z = nx$ $(m > n)$.

4. ~~Show that the volume of the solid obtained by revolving the cardioid $r = a(1 + cos\theta)$ about the initial line is $\frac{8\pi a^3}{3}$.~~

5. Use cylindrical coordinates to compute the integral $\iiint_D z(x^2 + y^2)^{-1/2} dxdydz$, where D is the solid bounded above by the plane $z = 2$ and below by the surface $2z = x^2 + y^2$.

6. The average value of a function f over a solid region D is defined as

$$\frac{1}{\text{vol. of D}} \iiint_D f(x, y, z) dxdydz.$$

   Find the average value of $f(x, y, z) = x + y + z$ over the sphere $x^2 + y^2 + z^2 = 4$.

7. Find the volume bounded above by the sphere $x^2 + y^2 + z^2 = 32$ and below by the paraboloid $x^2 + y^2 = 4z$.

8. Find the volume of the solid enclosed by the surfaces $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.

9. Find the volume bounded by the surfaces $z = 4 - x^2 - \frac{1}{4}y^2$ and $z = 3x^2 + \frac{y^2}{4}$.

10. Evaluate $\iiint z^2 dxdydz$ over the region common to the sphere $x^2 + y^2 + z^2 = a^2$ and the cylinder $x^2 + y^2 = ax$.

11. Find the centre of mass of the area bounded by the parabola $y^2 = x$ and the line $x + y = 2$, treating the density as constant.

12. Find the centre of mass of one complete arc of the cycloid: $x = a(t - sint)$, $y = a(1 - cost)$, treating the density as constant.

13. Find the mass of a plate in the shape of the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$, the density being given by $\rho = \mu xy$.

14. ~~A solid body of constant density ρ is obtained by revolving the cardioid $r = a(1 + cos\theta)$ about the initial line. Find its M.I. about a straight line through the pole and perpendicular to the initial line.~~

Answers.

1. $\frac{a^2}{2}(3\pi - 8)$
2. $8a^2$
3. $(m - n)a^3\pi$
5. $\frac{32\pi}{5}$
6. 0
7. $64\pi(\frac{4\sqrt{2}}{3} - \frac{7}{6})$
8. $\frac{16a^3}{3}$
9. $4\sqrt{2}\pi$
10. $\frac{2}{15}a^5\pi$
11. $(\frac{8}{5}, -\frac{1}{2})$
12. $(a\pi, \frac{4a}{3})$
13. $\frac{\mu a^2 b^2}{20}$
14. $\frac{352}{105}\rho a^5\pi$.