<u>Keys</u>

<u>Model Solution</u>

Indian Institute of Technology Roorkee
Mathematics I (MAI-101)
Quiz-II

Maximum marks: **10**

Autumn Semester 2024-25

Duration: **20 minutes**

Date: 18.11.2024

Name:...........................

Enrollment No.......................

Batch No.............

1. The value of

$$\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3} \, dy\,dx = \quad e-1$$

_____.

2. The value of

$$\int_{-1}^1 \int_{-\sqrt{1-y^2}}^0 \frac{4}{1+2(x^2+y^2)} dx\,dy = \quad \pi \ln 3$$

_____.

3. By changing the order of integration, put the remaining limits of integration on the right hand integral of the following

$$\int_0^1 \int_{-1}^0 \int_0^{y^2} dz\,dy\,dx = \int_0^1 \int_0^1 \int_{-1}^{-\sqrt{z}} dy\,dx\,dz$$

4. The volume of the solid whose base is the region in the xy-plane that is bounded by the curves $y = 4 - x^2$ and $y = 3$, while the top of the solid is bounded by the plane $z = x + 4$, is equal to _____. $16/3$

5. The value of the integral $\int_0^\infty \frac{x^4(1+x^5)}{(1+x)^{15}} dx = \quad \dfrac{1}{5005}$

6. Let $p > 1$, $q > 1$ and $\beta(.,.)$ denotes the beta function. Let

$$\beta(p, q) = m\beta(p, q-1) = n\beta(p-1, q).$$

If $m + n = \alpha - \dfrac{\alpha}{p+q-1}$, then $\alpha = \underline{1}$.

7. If the surface $5x^2 + 10yz - 9z + 14 = 0$ cuts the surface $ax^2 + by^3 = 5$ orthogonally at $(1, -1, 1)$, then the value of $a + b$ is _____. 1

8. The flux of $\vec{V}(x, y, z) = 2x\hat{i} + 4y\hat{j} - 3z\hat{k}$ through the surface of the portion of the plane $x + y + z = 1$ included in the first octant is $\dfrac{1}{2}$. [Hint: You may use the Gauss Divergence Theorem]

9. Let $\vec{r}(t) = \sin t\hat{i} + \cos t\hat{j} - \sqrt{3}t\hat{k}$, $0 \leq t \leq \pi$ be the space curve in the parametric form with parameter t. If the parametric form of the curve in parameter of arc length s, is $\vec{r}(s)$, then $\dfrac{s}{t} = $ _____. 2

10. Let $\vec{V}(x, y, z) = (3x^2 + az)\hat{i} + x^3\hat{j} + (3x + 3z^2)\hat{k}$ be a vector field in $\mathbb{R}^3$. If $\vec{V}$ is a conservative field, then the value of a is _____. 3