NAME:
ENROL. NO.:
TUTORIAL BATCH:

MAI-101:Mathematics I (QUIZ-1, Autumn Semester 2024-25)

Time: 20 minutes SET-A Total marks: 10

Note: Fill in the blanks within the space mentioned on this sheet only.

1. Let A be a matrix with characteristic polynomial $(\lambda-1)^3(\lambda-2)^2$, and $\operatorname{rank}(A-2I)=3$. If A is not diagonalizable, then the sum of all possible values of rank $(A-I)$ is ...7

2. Let A be a 5×5 matrix such that $\operatorname{rank}(A)=3$. If a linear system $AX=b$ is inconsistent and rank of augmented matrix $(A|b)$ is r, then the value of r is ...4

3. Let M be a 2×2 matrix having eigenvalues -1 and -2 with the corresponding eigenvectors $(1,\ -1)^T$ and $(1,\ 1)^T$, respectively. Then, the sum of all elements of M is ...-4

4. A matrix B is obtained by applying elementary row operations $R_2 \to R_2 - R_1$ followed by $R_3 \leftrightarrow R_1$ in a 3×3 matrix A. If $A = P^{-1}B$, then the trace of the matrix P is ...1

5. Let $u = \sin^{-1}\left(\dfrac{x+2y}{\sqrt{x^9+y^9}}\right)$. If $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \alpha\ f(u),\quad \alpha\in\mathbb{R}$, then the value of $\left(\alpha f(\frac{\pi}{6})\right)^2$ is ...49||2

6. If $x = u^2+v^2$ and $y = 2uv$, then the value of $\left(\dfrac{\partial(u,v)}{\partial(x,y)}\right)^2$ at $(x,y)=(3,1)$ is ...1/128

7. Let $f:\mathbb{R}^2\to\mathbb{R}$ be defined as $f(x,y) = \begin{cases} \dfrac{2x^3y}{x^2+y^2}; & (x,y)\neq(0,0) \\ \xi; & (x,y)=(0,0). \end{cases}$ If the function f is continuous on $\mathbb{R}^2$, then the value of $\dfrac{\partial(f_y)}{\partial x}$ at $(0,0)$ is ...2

8. Let $L(x,y)$ be the linear approximation of the function $f(x,y)=\log_e(xy)$ at $(1,1)$ in a rectangle R containing the point $(1,1)$. Then, the value of $L(3,2)$ is ...3

9. Let $z=z(s,t)$, $s=3u+4v$, $t=3u-4v$, and $z_{st}=z_{ts}$. If $\dfrac{\partial^2 z}{\partial v^2}+\alpha\dfrac{\partial^2 z}{\partial s\partial t} = \beta\dfrac{\partial^2 z}{\partial s^2}+\gamma\dfrac{\partial^2 z}{\partial t^2}$, $\alpha,\beta,\gamma\in\mathbb{R}$, then the value of $\alpha+\beta+\gamma$ is ...64

10. Let $f(x,y)=\alpha x^2+\beta y^2+\gamma x+4y+6, \alpha,\beta,\gamma\in\mathbb{R}$, has a saddle point at $(1,1)$. If $(-\infty,\ m)$ is the largest interval for the values of $\beta+\gamma$, then m is equal to ...-2