

Indian Institute of Technology Roorkee
Mathematics I (MAI-101)
Mid Term Examination

Maximum marks: **50**
Autumn Semester 2024-25

Duration: **90 minutes**
Date: 10.09.2024

All the nine questions are compulsory.

1. Find all the values of α , β , and δ for which the system of equations

$$\begin{aligned} \alpha x - 2y + z &= 13, \\ -2x + \beta y + 4z &= 11, \\ x + 4y - 5z &= \delta \end{aligned}$$

has (i) exactly one solution (ii) no solution. [6]

2. Applying the elementary row operations, find the inverse of

$$A = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}.$$

[4]

3. Let $A = \begin{pmatrix} 7 & 2 & -1 \\ 2 & 7 & 1 \\ -1 & 1 & \frac{17}{2} \end{pmatrix}$ whose characteristic polynomial has a factor $2\lambda^2 - 27\lambda + 81$. Find an orthogonal matrix P and a diagonal matrix D such that $P^T A P = D$. [7]

4. Let $A = \begin{pmatrix} 1 & \alpha & 3 \\ 3 & \beta & 3 \\ 6 & -6 & \gamma \end{pmatrix}$ with its eigenvalues $-2, -2, \delta (\neq -2)$. If A is diagonalizable, then find the values of α, β, γ , and δ . [5]

5. a) Let A be a 6×6 square matrix such that $A^T = A^2$. Find all possible eigenvalues of A including real and complex.

- b) Find the values of a and b such that the matrices A and B are similar, where

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 2 & a & 2 \\ 3 & 1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & b \end{bmatrix}. \quad [4+5]$$

6. Let $A = \begin{pmatrix} 6 & 6 & 4 \\ 3 & -5 & 3 \\ 1 & 9 & 3 \end{pmatrix}$ be such that $A^4 = \alpha A^2 + \beta A + \gamma I$, then find the values of $\alpha + \beta + \gamma$.

[6]

7. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a function defined by

$$f(x, y) = \begin{cases} |x| + y(\sin \frac{1}{x}), & \text{if } x \neq 0; \\ 0, & \text{if } x = 0. \end{cases}$$

Compute both the iterated/repeated limits at $(0, 0)$ and also find $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$, if they exist. [5]

8. Examine the continuity of the function

$$f(x, y) = \begin{cases} \frac{2x(x^3 - y^3)}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0); \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$$

at the origin $(0, 0)$. [4]

9. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be a function defined by

$$f(x, y) = \begin{cases} x^2 \sin \frac{1}{x} + y^2 \sin \frac{1}{y}, & \text{if } xy \neq 0; \\ x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \text{ and } y = 0; \\ y^2 \sin \frac{1}{y}, & \text{if } x = 0 \text{ and } y \neq 0; \\ 0, & \text{if } x = 0 = y. \end{cases}$$

[4]

Compute the partial derivatives $f_x(0, 0)$ and $f_y(1, 1)$, if they exist.

Solution 1

$$\left[\begin{array}{ccc|c} 1 & 4 & -5 & 8 \\ -2 & \beta & 4 & 11 \\ \alpha & -2 & 1 & 13 \end{array} \right] \xrightarrow{\text{After row operations}} \left[\begin{array}{ccc|c} 1 & 4 & -5 & 8 \\ 0 & \beta+8 & -6 & 11+2\delta \\ 0 & -2-4\alpha & 1+5\alpha & 13-\alpha\delta \end{array} \right] \quad \left(\frac{1}{2} \text{ marks} \right)$$

✓ for one more step

$\left(\frac{1}{2} \text{ marks} \right)$

Unique solutions:

- 1) $\alpha = -\frac{1}{2}, \beta \neq -8, \delta \in \mathbb{R}$
- 2) $\beta = -8, \alpha \neq -\frac{1}{2}, \delta \in \mathbb{R}$
- 3) $\beta \neq -8, \alpha \neq -\frac{1}{2}, \beta \neq \frac{4(1-4\alpha)}{1+5\alpha}$

$\left(2 \text{ marks} \right)$

No solutions:

- 1) $\beta = -8, \alpha = -\frac{1}{2}, \delta \in \mathbb{R}$
- 2) $\beta \neq -8, \alpha \neq -\frac{1}{2}, \beta = \frac{4(1-4\alpha)}{1+5\alpha}$

$\left(2 \text{ marks} \right)$

Alternate:

For unique solution
Showing $\det A \neq 0$

getting conditions on $\alpha \neq \beta$.

$\left(2 \text{ marks} \right)$

Question 2.

$A = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$. Find inverse using elementary row operations.

$$\begin{pmatrix} 3 & -1 & 1 & | & 1 & 0 & 0 \\ -15 & 6 & -5 & | & 0 & 1 & 0 \\ 5 & -2 & 2 & | & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_2' = R_2 + 5R_1} \begin{pmatrix} 3 & -1 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & 5 & 1 & 0 \\ 5 & -2 & 2 & | & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_3' = R_3 - 2R_1}$$

$$\begin{pmatrix} 3 & 0 & 1 & | & 6 & 1 & 0 \\ 0 & 1 & 0 & | & 5 & 1 & 0 \\ -1 & 0 & 0 & | & -2 & 0 & 1 \end{pmatrix} \xrightarrow{R_1' = R_1 + R_2} \begin{pmatrix} 3 & -1 & 1 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & 5 & 1 & 0 \\ -1 & 0 & 0 & | & -2 & 0 & 1 \end{pmatrix} \xrightarrow{R_1' = R_1 + 2R_3}$$

$$\begin{pmatrix} 1 & 0 & 1 & | & 2 & 1 & 2 \\ 0 & 1 & 0 & | & 5 & 1 & 0 \\ -1 & 0 & 0 & | & -2 & 0 & 1 \end{pmatrix} \xrightarrow{R_3' = R_3 + R_1} \begin{pmatrix} 1 & 0 & 1 & | & 2 & 1 & 2 \\ 0 & 1 & 0 & | & 5 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 1 & 3 \end{pmatrix} \xrightarrow{R_1' = R_1 - R_3}$$

$$\begin{pmatrix} 1 & 0 & 0 & | & 2 & 0 & -1 \\ 0 & 1 & 0 & | & 5 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & 1 & 3 \end{pmatrix} \xrightarrow{R_1' = R_1 - R_3}$$

Full marks : 4

~~Per row correct~~

1) A^{-1} is determined correctly : 4

2) A^{-1} " not " : 1 mark for each correct row.

PTC



3) None of the rows are correct but performed at least ~~203~~ ³⁻⁵

row operations correctly : 1 mark,

4) One row correct + 4-5 operations correct : 2

5) Only one or two digits wrong : 3 (or 3.5 if the mistake is made at the last/penultimate step)

6) { One row correct + almost all steps are correct : $2\frac{1}{2}$
Two rows correct + 4-5 operations correct.

$$A = \begin{pmatrix} 7 & 2 & -1 \\ 2 & 7 & 1 \\ -1 & 1 & 17/2 \end{pmatrix}$$

$$2\lambda^2 - 27\lambda + 81 = 0 \Rightarrow (2\lambda - 9)(\lambda - 9) = 0 \Rightarrow \lambda = 9, 9/2 \quad \text{--- (1)}$$

$$\text{trace}(A) = 7 + 7 + 17/2 = 9 + 9/2 + x \Rightarrow x = 9 \quad \text{--- (1)}$$

Hence, eigenvalues of A are $9, 9, 9/2$.

$$(A - 9I)X = 0 \Rightarrow \begin{pmatrix} -2 & 2 & -1 \\ 2 & -2 & 1 \\ -1 & 1 & -1/2 \end{pmatrix} X = 0 \Rightarrow \begin{pmatrix} -2 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} X = 0$$

$$\Rightarrow -2x_1 + 2x_2 - x_3 = 0 \quad \text{--- (1)}$$

$$X = \begin{pmatrix} -1 & 0 & 2 \end{pmatrix}^T; \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}^T \text{ LI e-vectors for } \lambda = 9.$$

$$(A - \frac{9}{2}I)X = 0 \Rightarrow \begin{pmatrix} 5/2 & 2 & -1 \\ 2 & 5/2 & 1 \\ -1 & 1 & 4 \end{pmatrix} X = 0 \Rightarrow \begin{pmatrix} -1 & 1 & 4 \\ 2 & 5/2 & 1 \\ 5/2 & 2 & -1 \end{pmatrix} X = 0$$

$$\Rightarrow \begin{pmatrix} -1 & 1 & 4 \\ 0 & 9/2 & 9 \\ 0 & 0 & 0 \end{pmatrix} X = 0$$

$$\Rightarrow \begin{cases} -x_1 + x_2 + 4x_3 = 0; & x_2 + 2x_3 = 0 \end{cases} \quad \text{--- (1)}$$

or $x_1 = 2\alpha, x_2 = -2\alpha, x_3 = \alpha, \alpha \in \mathbb{R}$

$$X = \begin{pmatrix} -2 & 2 & -1 \end{pmatrix}^T \text{ LI e-vector for } \lambda = 9/2. \quad \text{--- (1)}$$

clearly $v_1^T v_2 \neq 0$.

Taking v_1, v_3 and $v_2' = (a, b, c)^T$, we get

$$-a + 2c = 0; -2a + 2b - c = 0 \Rightarrow v_2' = (4 \ 5 \ 2)^T \quad \text{--- (1)}$$

$$\text{So, } P = \begin{pmatrix} -1/\sqrt{5} & 4/\sqrt{45} & -2/3 \\ 0 & 5/\sqrt{45} & 2/3 \\ 2/\sqrt{5} & 2/\sqrt{45} & -1/3 \end{pmatrix}; D = \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9/2 \end{pmatrix} \quad PP^T = I$$

Taking v_2, v_3 and $v_1' = (a, b, c)^T$, we get

$$a + b = 0; -2a + 2b - c = 0 \Rightarrow v_1' = (-1 \ 1 \ 4)^T \quad \text{--- (1)}$$

$$\text{So, } P = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{18} & -2/3 \\ 1/\sqrt{2} & 1/\sqrt{18} & 2/3 \\ 0 & 4/\sqrt{18} & -1/3 \end{pmatrix}; D = \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9/2 \end{pmatrix} \quad PP^T = I$$

Q.No.4:

Since A is diagonalizable, hence

$$\text{Rank}(A+2I) = 1.$$

(1M)

$$\Rightarrow A+2I = \begin{bmatrix} 3 & \alpha & 3 \\ 3 & \beta+2 & 3 \\ 6 & -6 & \gamma+2 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - R_1$$

$$R_3 \rightarrow R_3 - 2R_1$$

$$\Rightarrow \begin{bmatrix} 3 & \alpha & 3 \\ 0 & \beta+2-\alpha & 0 \\ 0 & -6-2\alpha & \gamma-4 \end{bmatrix}$$

(2M)

$\text{Rank}(A+2I) = 1$ gives

$$\boxed{\alpha = -3}$$

$$\boxed{\gamma = 4}$$

$$\boxed{\beta = -5}$$

(1M)

Now Trace $A = 1 + \beta + \gamma = 0$

Hence $\boxed{S = 0 + 2 + 2 = 4}$

(1M)

Q5.

① Let A be a 6×6 square matrix such that $A^T = A^2$. Find all possible eigenvalues of A including real and complex.

Since, it is given that A is a 6×6 square matrix,

$$\Rightarrow A \in \mathbb{C}^{6 \times 6}$$

Also, $A^2 = A^T \quad \text{--- (1)}$

Taking transpose on both side, we get

$$(A^2)^T = (A^T)^T$$

$$\Rightarrow (A^T)^2 = (A^T)^T$$

$$\Rightarrow (A^2)^2 = A \quad (\text{using (1)})$$

$$\Rightarrow A^4 = A \quad \text{or} \quad A^4 - A = 0 \quad \text{--- (2)}$$

Egⁿ (2) shows $f(x) = x^4 - x$ is an annihilating polynomial of A .

Therefore, roots of $f(x) = x^4 - x$ are the possible eigen values of A

$$\Rightarrow f(x) = 0 \Rightarrow x^4 - x = 0$$

$$\Rightarrow x(x^3 - 1) = 0$$

$$\Rightarrow x = 0, 1, \omega, \omega^2$$

Where ω is cube roots of unity

Hence, Possible eigen values of A are $0, 1, \omega, \omega^2$

④ marks

OR

Given $A^2 = A^T$

Let d be an eigen value of A , Then

For given ($x \neq 0$) $Ax = dx \quad (x \neq 0)$

taking transpose on both side

$$(Ax)^T = (dx)^T$$

$$\Rightarrow x^T A^T = d x^T$$

Multiplying x on right side, we get

$$\Rightarrow x^T A^T x = d x^T x$$

$$\Rightarrow x^T A^2 x = d x^T x$$

$$\Rightarrow x^T A(Ax) = d x^T x$$

$$\Rightarrow x^T A(dx) = d x^T x$$

$$\Rightarrow x^T d(Ax) = d x^T x$$

$$\Rightarrow x^T d(dx) = d x^T x$$

$$\Rightarrow x^T d^2 x = d x^T x$$

$$\Rightarrow d^2 x^T x = d x^T x$$

$$\Rightarrow (d^2 - d) x^T x = 0 \quad (x^T x \neq 0)$$

$$\Rightarrow d^2 - d = 0 \Rightarrow d = 0, 1 \quad \text{② marks}$$

Q5
(b) Find the values of a and b such that the matrices A and B are similar, where

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 2 & a & 2 \\ 3 & 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & b \end{bmatrix}$$

Sol.

Given A and B are similar, Therefore $\exists$ an invertible matrix P such that

$$B = P^{-1}AP$$

Also, By the properties of similar matrices,

A and B have same rank, same trace, same determinant, also same eigen values, with same algebraic and geometric multiplicity.

Therefore, $\text{Trace}(A) = \text{Trace}(B)$

$$\Rightarrow -2 + a + 1 = -1 + 2 + b$$

$$\Rightarrow a - 1 = b + 1$$

$$\Rightarrow \boxed{a - b = 2} \quad \text{--- (1)} \quad \text{(2) marks}$$

Also, Eigen values of B are its diagonal elements (As B is a diagonal matrix)

$$\therefore \lambda = -1, 2, \text{ and } b$$

Since, A and B are similar, implies $-1, 2$ and b are eigen values of A as well.

$$\text{Therefore, } |A - (-1)I| = 0 \quad (\text{i.e. } \det(A - (-1)I) = 0)$$

$$\Rightarrow |A + I| = 0$$

$$\Rightarrow \begin{vmatrix} -2+1 & 0 & 0 \\ 2 & a+1 & 2 \\ 3 & 1 & 1+1 \end{vmatrix} = 0$$

$$\Rightarrow -1 \begin{vmatrix} a+1 & 2 \\ 1 & 2 \end{vmatrix} = 0$$

$$\Rightarrow -[2(a+1) - 2] = 0$$

$$\Rightarrow -2a - 2 + 2 = 0$$

$$\Rightarrow \boxed{a = 0} \quad \text{--- (2) marks} \quad \text{--- (2) marks}$$

Using eqn (2) in eqn (1) we get

$$0 - b = 2 \Rightarrow \boxed{b = -2} \quad \text{--- (3)} \quad \text{(1) marks}$$

$$\textcircled{6} |A - \lambda I| = \begin{vmatrix} 6-\lambda & 6 & 4 \\ 3 & -5-\lambda & 3 \\ 1 & 9 & 3-\lambda \end{vmatrix}$$

$$= (6-\lambda)[-(5+\lambda)(3-\lambda)-27]-6(9-3\lambda-3)+4(27+5+\lambda)$$

$$= (6-\lambda)[-(-\lambda^2-2\lambda+15)-27]+18\lambda-36+4\lambda+128$$

$$= (6-\lambda)[\lambda^2+2\lambda-42]+22\lambda+92$$

$$= -\lambda^3-2\lambda^2+42\lambda+6\lambda^2+12\lambda-252+22\lambda+92$$

$$= -\lambda^3+4\lambda^2+76\lambda-160$$

So the characteristic equation of A is

$$-\lambda^3+4\lambda^2+76\lambda-160=0$$

[3]

So by the Cayley-Hamilton Theorem

$$A^3 = 4A^2 + 76A - 160I$$

[1]

$$\Rightarrow A^4 = 4A^3 + 76A^2 - 160A$$

$$= 4[4A^2 + 76A - 160I] + 76A^2 - 160A$$

$$= 92A^2 + 144A - 640I$$

[1]

$$\Rightarrow \alpha + \beta + \gamma = 92 + 144 - 640$$

$$= -404$$

[1]

Question 7:

$$f(x,y) = |x| + y \sin \frac{1}{x}$$

$\lim_{y \rightarrow 0} \lim_{x \rightarrow 0} f(x,y)$ does not exist with^a justification saying 1
that $\sin \frac{1}{x}$ does not behave well if $x \rightarrow 0$. 2

without justification. 1/2

$$\lim_{x \rightarrow 0} \lim_{y \rightarrow 0} f(x,y) = 0 \quad \text{2}$$

$$\lim_{(x,y) \rightarrow (0,0)} f(x,y) = 0$$

1) Complete answer with proper ϵ - δ method 2

2) with ϵ, δ implications, but inequalities are correct 1/2

3) Just answer. 1

Q.8. To Examine the continuity of $f(x,y) = \begin{cases} \frac{2x(x^3-y^3)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$

SOLUTION: As $(x,y) \rightarrow (0,0)$, Numerator has more degrees of smallness than denominator. Hence to evaluate $\lim_{(x,y) \rightarrow (0,0)} |f(x,y) - 0|$.

METHOD-I: $x = r \cos \theta$, $y = r \sin \theta$. Then $|f(x,y)| = |f(r \cos \theta, r \sin \theta)| = |2r^2(\cos^3 \theta - \sin^3 \theta)|$
 $|2r^2(\cos^3 \theta - \sin^3 \theta)| \leq 2r^2(|\cos^3 \theta| + |\sin^3 \theta|) \leq 4r^2$ (OR) $|2r^2(\cos^3 \theta - \sin^3 \theta)| \leq 2r^2(|\cos \theta - \sin \theta|)(1 + |\sin \theta \cos \theta|)$
 $\leq 2r^2 \sqrt{2} (3/2) = 3\sqrt{2}r^2$
 using $r \leq \delta$ gives $4\delta^2 \leq 4\delta^2 < \epsilon$ (OR) using $r \leq \delta$, $3\sqrt{2}\delta^2 \leq 3\sqrt{2}\delta^2 < \epsilon$

Hence given $\epsilon > 0$, $\exists \delta > 0$ s.t. $|f(x,y)| < \epsilon$ whenever $0 < r < \delta$

NOTE: For $re^{i\theta}$, if modulus is missing only ZERO mark

METHOD-II $|f(x,y)| = \left| \frac{2x(x^3-y^3)}{x^2+y^2} \right| \leq \left| \frac{2x^4}{x^2+y^2} \right| + \left| \frac{2xy^3}{x^2+y^2} \right|$. Now,

$$\left| \frac{2x^4}{x^2+y^2} \right| + \left| \frac{2xy^3}{x^2+y^2} \right| = \left| 2 \frac{x^2}{x^2+y^2} x^2 \right| + \left| \frac{2xy \cdot x^2}{x^2+y^2} \right|$$

(which using $x^2 \leq x^2+y^2$ and $2xy \leq x^2+y^2$) (OR) (using $x^2 \leq x^2+y^2$, $y^2 \leq x^2+y^2$)

$$\begin{aligned} &\leq 2|x|^2 + |y|^2 \leq 2|x|^2 + |y|^2 \leq 2|x^2+y^2| + |x^2+y^2| = 3|x^2+y^2| \\ &\leq 2\delta_1^2 + \delta_2^2 \leq 2\delta^2 + \delta^2 = 3\delta^2 \leq \epsilon \end{aligned}$$

where $\delta = \max\{\delta_1, \delta_2\}$ where, $0 \leq \sqrt{x^2+y^2} \leq \delta$

$|x| \leq \delta_1$, $|y| \leq \delta_2$ (Rectangular neighbourhood) (Circular neighbourhood)

Hence, given $\epsilon > 0$, $\exists \delta > 0$ such that $|f(x,y)| < \epsilon$

whenever $0 \leq \sqrt{x^2+y^2} \leq \delta$ (OR) $|x| \leq \delta_1$, $|y| \leq \delta_2$ with $\delta = \max\{\delta_1, \delta_2\}$,
 where $3\delta^2 \leq \epsilon$

NOTE: By other inequalities, one may get $6\delta^2 \leq \epsilon$.

This gives $3\delta^2 \leq \epsilon$, $3\sqrt{2}\delta^2 \leq \epsilon$, $4\delta^2 \leq \epsilon$, $6\delta^2 \leq \epsilon$

Any other option Zero marks

DIRECTLY getting one of these four inequalities, with proper procedure
 4 marks

NOTE: $\rightarrow$ Any other inequality ZERO marks.

$\rightarrow$ Modulus missing ZERO marks

$\rightarrow$ Using $y = mx$ method ONE mark.

Solution-9

Solⁿ:

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{f(0+h, 0) - f(0, 0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{h^2 \sin \frac{1}{h} - 0}{h} \\ &= \lim_{h \rightarrow 0} h \sin \frac{1}{h} = 0 \end{aligned}$$

$\therefore f_x(0, 0) = 0.$ (+1)

Now

$$\begin{aligned} & \lim_{k \rightarrow 0} \frac{f(1, 1+k) - f(1, 1)}{k} \\ &= \lim_{k \rightarrow 0} \frac{(1+k)^2 \sin \frac{1}{1+k} - \sin 1}{k} \quad \text{(+1)} \\ &= \lim_{k \rightarrow 0} \left[\frac{(\sin \frac{1}{1+k} - \sin 1)}{k} + (2+k) \sin \frac{1}{1+k} \right] \\ &= \lim_{k \rightarrow 0} \left[\frac{(-2) \cos \frac{2+k}{2(1+k)} \sin \frac{k}{2(1+k)}}{k} + (2+k) \sin \frac{1}{1+k} \right] \\ &= \lim_{k \rightarrow 0} \left[(-2) \cos \frac{2+k}{2(1+k)} \cdot \frac{\sin \frac{k}{2(1+k)}}{\frac{k}{2(1+k)}} \cdot \frac{1}{2(1+k)} + (2+k) \sin \frac{1}{1+k} \right] \quad \text{(+1)} \\ &= -\cos 1 + 2 \sin 1 \end{aligned}$$

as $\lim_{\frac{k}{2(1+k)} \rightarrow 0} \frac{\sin \frac{k}{2(1+k)}}{\frac{k}{2(1+k)}} = 1.$

Therefore $f_y(1, 1) = 2 \sin 1 - \cos 1.$ (+1)